OPTIMIZING COST AND REVENUE FOR SOLID WASTE MANAGEMENT IN NYAMIRA MUNICIPALITY USING FUZZY GOAL PROGRAMMING MATHEMATICAL MODEL

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DECLARATION AND APPROVAL

Declaration

I declare that this thesis is my original work and has not been presented for the conferment of a degree or award of diploma in this or any other university.

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PGC/AM/003/20

Approval

This thesis has been submitted with our approval as the University Supervisors.

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DEDICATION

I dedicate this thesis to my son Kylian Hannington Okumu, my dear wife Bevaline Chelangat, my parents Peter Khafulu and Cisciliah Andiega without forgetting my siblings Godliver Bwire, Evans Khafulu Dennis Bwire Felistus Akelo and Roseline Auma.

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ABSTRACT

Waste in urban areas is growing rapidly everywhere in the world. Effective methods to address challenges of solid waste management are critical in achieving an environment that is clean and healthy. The problem of waste growth in urban areas has been brought about by low collection of waste, illegal and uncontrolled dumping sites and the absence of sewer lines. In Kenya for instance the growth of economy has resulted in the development of cities and emergence of towns leading to waste management challenges. Efforts have been directed towards addressing the problem of waste management by various county governments. This study sought to address solid waste disposal and management challenges in Nyamira Municipality. The methods which are being employed to address this challenge was costly and had not yielded desirable results as was evident with the scattered waste in the streets of Nyamira Municipality. Currently the management does not carry out any waste recycling and has hired one landfill which is paid for monthly. This Study formulated solid waste management tool that involved construction of a fuzzy goal programming model with two objective functions. The model was solved analytically by simplex method. The obtained results for first objective function of minimizing the cost of solid waste management is ksh. 26,557,125 which gives a membership function of 0.6 and the second objective function of maximizing revenue is ksh 9,500,000 that is 0.5 membership function. This findings has reduced the net system cost of solid waste management in Nyamira Municipality by 36%. Numerical simulation and sensitivity analysis was carried by varying the operation cost of recycling facility and on the capacity of recycling facility parameters and then the graphs drawn using MATLAB software. The utility of the model was tested using data from Nyamira municipality. The research advocates for waste recycling as one of the ways of managing waste so as to earn revenue from these recycled waste. The findings of this study is useful in formulating policies such setting up of waste management projects and establishment of recycling industries. Furthermore the findings forms the basis for future research in related fields.

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LIST OF ABBREVIATIONS

CCP	Chance-Constrained Programming
FGP	Fuzzy Goal Programming
GHG	Greenhouse Gases
GP	Goal Programming
MCDA	Multi Criteria Decision Analysis
MSW	Municipal Solid Waste
MSWM	Municipal Solid Waste Management
PE	Poly Enthylelene
PET	Poly Enthyle Terephthalate
PP	Poly Prophylene
PS	Poly Styrene
PVC	Poly Vinyl Chloride
SWMS	Solid Waste Management System
SWM	Solid Waste Management

LIST OF SYMBOLS/NOTATIONS

$\mu_A(x)$	Membership function
F_i	Level of achievement
g_i	Aspiration level
X	Universe of discourse set
d_i^+	Overachievement
d_i^-	Underachievement
Z^k	k^{th} objective function
c_{ij}^k	Charges on waste management
$\in$	Element of
$\notin$	Not element of

DEFINITION OF TERMS

Basic variables

Basic variables are non negative parameters contained in the optimal solution (Laurel and Gina 2015)

Constraints

Constraints are a group of inequalities and equalities that are a set of conditions to be satisfied when computing an optimal solution (Laurel and Gina 2015) .

Linear program

Linear program is a method that is used to attain the optimal solution having maximizing or minimizing equations whose constraints are linear (Laurel and Gina 2015) .

Mathematical programming model

Mathematical programming is mathematical model that is used to find solutions for problems whose transformations are linear. This model applies the criteria of optimization (Mula *et al* 2010)

Membership function

Membership function is defined as a method of finding a solution to a problem by experience instead of knowledge by mapping $m : X \rightarrow [0,1]$ (Perycz 1994).

Non-basic variables

Non-basic variables are zero parameters in condition of the best solution (Laurel and Gina 2015).

Objective function

Objective function is mathematical equation of decision variable that is to be optimized by either maximizing or minimizing in the process of attaining the optimal solution (Rafiei *et al* 2013) .

Optimization

Optimization is a mathematical technique for finding a minimum or maximum functional value of many parameters subjected to a set of constraints (Laurel and Gina 2015) .

Optimal solution

An optimal solution is the value given to the parameters in the function to be maximized or minimized to get best value (Laurel and Gina 2015).

Pivot variable

Pivot variable is a variable used operation in the rows in identifying the variable that the unit value will become and is an important parameter in the unit value conversion (Laurel and Gina 2015).

Simplex method

Simplex method is an analytical method for finding solution for programming models that are linear by use of pivot variables, tableaus and slack variables as a way of looking for the optimization problem's solution that is desired (Laurel and Gina 2015)

Simplex tableau

Simplex tableau is a table that is used to check optimality by performing row operations on the linear programming model (Laurel and Gina 2015) .

Slack variables

Slack variables are parameters which are added to constraints to convert the constraints from an inequality to equality in a linear program (Laurel and Gina 2015) . Standard form Standard form is all the linear programs baseline format before finding solution that is optimal (Laurel and Gina 2015).

Waste

Waste is unwanted materials or substance which is disposed off after primary use or is regarded as worthless, defective and of no use (Amasuomo 2016).

CHAPTER ONE

INTRODUCTION

1.1 Overview

This chapter highlights the background of the study, statement of the problem, objectives of the study, significance, assumptions and the scope of the study.

1.2 Background of the study

1.2.1 Nyamira Municipality

Nyamira municipality is located in Nyamira county in the former Nyanza province of Kenya. It is the largest town in Nyamira county as well as its headquarters, with a population of 24483 persons (KNBS 2019 census). The municipality is located in township ward, Nyamatuta Chache Location. The Municipality consists of Township and Siamai Sub-locations. Other areas that contribute to Nyamira Municipal waste include Bigege Sub-location with a population of 10,579 persons and Ikobe Sub-loacation with a population of 7,882 persons (KNBS 2019). Nyamira Municipality and its periphery generated a lot of solid waste which was being poorly managed. Solid waste generation in municipality was from households, schools, markets and hospitals. The main waste management was through collection and dumping in designated dumpsite.

In Nyamira municipality the rise in population growth and the development of the town, escalation and establishment of more commercial and service activities had led to production of a lot of solid waste in the municipality. As a result,this has led to problems such as sewerage blockages and environmental pollution. To overcome these challenges has strained the budget of the county government.

1.2.2 Municipal Solid Waste (MSW)

Municipal Solid Waste is an unavoidable by-product of human activity that comprises all wastes procreated within the territory of municipality. It may also be referred to as waste produced, gathered, transported and discarded within the boundaries of the municipal.

According to Hoornweg et al. (2012) Municipal Solid Waste is described as waste gathered and discarded by the authority of the municipal at the municipal dumpsites. This includes wastes from residences, industries, institutions, commercial centers and construction and demolition sites. In many cases municipal solid waste constitutes food remnants and garbage from residences, street sweeping.

According to Zhou et al. (2014) Municipal solid wastes are categorized into ; food remnants, plant, paper, textile and rubber. Food remnants in MSW is the food appropriate for human consumption being discarded whether or not after it is kept beyond its expiry date or left to spoil. It includes bones, peels from fruits, vegetables wastes, cone wastes and nutshells from nuts among others. Plant waste is waste that is derived from plant source that remain to decompose or be burned. This kind of waste in MSW include wood, bamboo, leaves, weeds among others. Paper is any form of paper that becomes unusable and needs disposing. This include newspapers, books, printing papers, cardboard so and toilet papers. Textile waste is materials that are no longer usable or are considered worthless after end of production process of any textile product. This include fibers, wool and cotton. Municipal solid waste from plastics is different from other wastes in that it exists in varieties which are pure. They include poly ethylene (PE), poly prophylene (PP) PS, PVC and poly ethylene terephthalate (PET). Rubber waste is solid waste that consists of petroleum based products. Examples are belts and tires.

1.2.3 Solid Waste Management (SWM)

Solid Waste Management is an expression that is used to refer to the task of gathering, treating and dumping waste that is solid in nature. Solid Waste Management (SWM) is also described as a set of systematic and consistent rules relating to the control of generating, keeping, collecting, transporting, procession of waste and waste land-filling to the best principles of public health, conservation of resources, aesthetics, economy, other requirements of the environment and what the public requires.

Pattnaik and Reddy (2010) described Solid waste management as effective and efficient collecting, transporting, and disposing of waste coming from residences, sweepings from streets, wastes from construction sites, non- dangerous, industries and imports including secondhand cloths popularly known as *mitumba* in Kenya. Solid waste management means a sequence of activities covering all functional elements including; generation of waste, handling, separating, storing, and processing at the generation point, collection, separating, processing and transformation at treatment facilities and final disposal. There are various methods of solid waste management including open burning, sea dumping, sanitary land filling, incineration, composing among others.

1.2.4 Mathematical Modeling

Mathematical modeling is termed as the process of developing mathematical models. Modeling in mathematics involves the conversion of real world problem such as solid waste management into a problem that can be solved mathematically by use of equations and mathematical symbols. Mathematical modeling can also be described as conversion activity problem which is real into a mathematical form.

Galbraith & Clatworthy (1990) described Mathematical modeling as the mathematical application in the finding of the solution for problems which are not structured in real-life situations. In modeling, approaches that are mathematical are used in solving challenges related to problems in real-life situations. Cheng et al. (2001) argues that real-life problems that we come across are converted into mathematical problems and solved using mathematical technique. According to Sarakikya et al.(2020) mathematical model consists of governing equation, assumptions and constraints, initial as well as boundary conditions. Various classifications on conditions can be used for mathematical models depending on their structure. It is significant to derive equations so that their differences and similarities are pointed out and reflected on for possible implications in their implementation to mathematical modeling. The methods of Mathematical modeling produce a virtual reality which when applied, may populate with everything that moves, irrespective of scale. Mathematical modeling allows the user to carry out experiments that in the real life are difficult, expensive and dangerous or impossible to measure.

1.2.5 Fuzzy Set

Definition

Fuzzy sets are sets with degree of membership for its elements. Fuzzy set is an ordered pair (X,m) where X is a non empty set known as universe of discourse and m is a mapping $m\colon X \to$ [0,1] (Zadeh 1965,)

For each $x \in X$ the value of $m(x)$ the degree of membership of $x(X,m)$

where

$m = \mu_A(x)$ is a membership function of the fuzzy set A defined by

$$\mu_A(x) = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ if } x \notin A \end{cases} \tag{1.1}$$

Therefore

$$\mu_A(x) \in [0,1] \tag{1.2}$$

For all $x \in X$, then

i) x is not inclusive in the fuzzy set A if $m(x) = 0$

ii) x is partially inclusive in the fuzzy set A if $0 < m(x) < 1$

iii) x is fully inclusive in the fuzzy set A if $m(x) = 1$

1.2.6 Triangular membership function

There are various membership functions including triangular, trapezoidal, piecewise e.t.c. The triangular membership function is distinguished by three variables a,b,c where a, b and c represent the coordinates of X for the three vertices in a fuzzy set A of $\mu_A(x)$ (a is the lowest limit and c is upper limit where membership degree is zero and b is the centre where membership degree is 1). Therefore

$$\mu_A(x) = \left\{ \begin{array}{cc} 0 & \text{if } x \leq a \\ \frac{x-a}{b-a} & \text{if } a \leq x \leq b \\ \frac{c-x}{c-b} & \text{if b} \leq \text{x} \leq \text{c} \\ 0 & \text{if } c \leq x \end{array} \right\} \tag{1.3}$$

1.2.7 Goal Programming

Watada J et al (2022) Goal Programming is a branch of multi-objective optimization. It involves multi-criteria decision analysis (MCDA). It is generalization of linear programming in the sense that it can handle multiple variables that are conflicting in nature. It can be used to solve the conflicting aspiration levels in terms of minimizing cost and maximizing profit. The deviations in the target of achievement defined in terms of minimizing cost and maximizing profit are calculated. The deviation from the set targets of achievement are then minimized to the satisfaction of the decision maker(s). Goal programming was used first by Cooper, Charnes and Ferguson in 1955. The following three types of analysis are performed using goal programming

i) Determine the amount of resources needed to achieve a set of objectives that are desired.

ii) Determine the attainment degree of the set objectives using the resources available.

iii)Provision of the best solution under different circumstances such as shifting priorities of the goals and changing amount of resources.

The goal programming formulations orders the deviations into various levels of priority, with the minimizing of the deviation in higher levels of priority being much more important than any other deviations in lower levels of priority. Formulating the GP models involves the following steps.

i) Defining the decision variables.

ii) Stating the constraints.

iii)Determining the preemptive priorities.

iv) Determining the relative weights.

v) Stating the objective function.

The overall objective of GP is the minimizing of deviations that arises among the levels of attainment of goals and their levels of aspiration. Charnes and Cooper (1977) the GP is expressed as follows

$$\sum_{i=1}^{k} |\ \mathrm{F_i}(\mathrm{x}) - \mathrm{g_i}| \qquad (1.4)$$

Subject to the following

$X = \{x \in \mathbb{R}^n\}$

Here $\mathrm{F_i}(\mathrm{x})$ is the linear functional achievement of the i^{th} goal and $\mathrm{g_i}$ is the level of aspiration of i^{th} goal.

Therefore

$$F_i(x) - g_i = d_i^+ - d_i^- \text{ for } d_i^+, d_i^- \geq 0 \tag{1.5}$$

Therefore, the GP be formulated as follows:

$$\sum_{i=1}^{k} |F_i(x) - g_i| = \sum_{i=1}^{k} |\ \mathrm{d_i^+} - \mathrm{d_i^-}\ | \tag{1.6}$$

Subject to the following conditions

$$\begin{aligned} &F_i(x) - d_i^+ + d_i^- - g_i = 0_i i = 1, 2, \ldots k \\ &X = \{x \in \mathbb{R}^n\} \end{aligned} \tag{1.7}$$

1.2.8 Fuzzy Goal Programming

Fuzzy goal programming is an extension of convectional goal programming for solving problems which have more than one objective functions with unclear defined model parameters in the environment for making decision. In this extension of convectional goal programming, the aspiration levels of every objective function is taken as a unity concerning attainment of the highest degree (unity) of goals of the problem which are naturally fuzzy. The element is said to be approaching full membership in A if it is close to 1 which is its highest degree of attainment.A fuzzy set A could be assigned by a membership occasionally symbolized by μ that maps every domain object of its membership grade in A (Zadeh 1965, Lotfi 2014).

Belmokaddem et al (2009), various types of membership functions exist that are used to promote the fuzzy analytical framework whose description of fuzzy is conjectural and the values of membership are subjective. This membership functions (MF) which are used in different analysis includes types such as exponential, linear, piecewise linear function and hyperbolic function. Generally, the linear membership which are neither increasing nor decreasing are mostly applied in the inequalities which are less than / equal to and greater than / equal to respectively to relationship.

According to Zimmerman (1978), the procedure for solving fuzzy goal programming involves satisfying the fuzzy objectives and the decision in the fuzzy situation is therefore defined at the junction of fuzzy objectives corresponding to those membership functions, Zimmerman (1985). The decision that is optimal can be any option in such decision marking environment that can be optimized by maximizing the minimum whose representation is by the corresponding membership functions. Simultaneous solution of complex objective system is allowed in this approach and establishment among various objectives is required for problem's solution. The main concept for linear goal programming is to turn into specific numeric goal from original multiple objectives for every objective. Usually Goal Programming models contains three components, these are; non negativity requirements, objective functions and goal constraint sets.

1.3 Statement of the Problem

As one walks around Nyamira Municipality scattered solid wastes is seen in the streets as shown in figure 6.1. This can be as a result of poor management. These wastes require large size of land which is not available in the municipality since it even rely on private owned landfill of which it is charged monthly which has further increased the cost of solid waste management. The available dumpsite is almost full as shown in figures 6.2 and 6.3. This calls for a mechanism to reduce the amount of waste ending up at the landfill using a method that is cost effective and does not pollute the environment. One of these ways of reducing waste is by recycling which in return will generate revenue. As a result a fuzzy goal programming mathematical model was found to be effective in addressing this challenge of Solid waste management in Nyamira Municipality.

1.4 Objectives of the Study

1.4.1 General Objectives

To develop an optimizing cost and revenue for solid waste management in Nyamira Municipality using fuzzy goal programming mathematical model.

1.4.2 Specific Objectives

i) To formulate FGP solid waste management model.

ii) To determine optimum solution from FGP solid waste management model the model.

iii) To perform numerical simulation.

iv) To carry out sensitivity analysis on the FGP solid waste management model.

1.5 Significance of the Study

The findings of this study can be employed in developing and setting up of waste management projects other than recycling facilities by the authority of the municipality. These projects may include energy production from waste, incineration among others which will go a long way in reducing waste reaching landfill.

The model promotes recycling of the waste, this can help in establishment of some industries such as shoe making from waste tires, bottle water manufacturing due to reuse of plastic bottles, tissue manufacturing from waste papers, fertilizer manufacturing among others. The products from these industries will generate income to people and revenue to the government. The projects will create employment both direct e.g. scavengers as well as indirect such those who will be employed to work in established industries.

1.6 Justification of the Study

Every human deserves to stay in a clean and healthy environment. Therefore, every county government needs to transit from uncontrolled dumpsites, illegal dumping and low waste collection rates to cheap waste collection cost and recycling in the best way possible in Nyamira Municipality. The best way is the one that is traceable, measurable and able to predict scientifically the trend. Therefore, fuzzy goal programming is a scientific approach which when employed in management of solid waste can yield desirable results such as solid waste management reduced cost and also able to generate employment and income through recycling.

1.7 Scope of the Study and Limitation

Within the model, the study was confined to the concept of cost and profit from recycling. Focus was on proper and efficient process of solid waste management based on the argument that transport cost and operating cost are among the factors to measure the performance and therefore efforts concentrating on enhancing good designs are likely to result in a notable attainment of solid waste management. The study was conducted in Nyamira municipal and the required information was obtained from environmental department.

1.8 Assumptions

1. The sorting was done at the collection centre to separate the waste to be disposed of and those to be recycled.
2. The capacities such as; landfill facility capacity, recycling facility capacity and collection facility capacity were limited.
3. All waste generated must be removed from the collection center
4.) The decision maker determines the prices of the recycled waste

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

In this chapter review of the existing literature will be undertaken highlighting the contributions of various scholars in this area of study. Conceptual framework of the model to be formulated will be discussed and the knowledge gap that is required to be filled will be identified.

2.2 Review of Related Literature

Shaban et al. (2022) formulated a mixed integer linear programming (MILP) model for solid waste management that integrated generation of waste, collection of waste, transfer of waste, recycling projects, incinerators and landfills. The model aimed at determining the locations and optimal number of different facilities and the flows of wastes in that the daily cost in the system is minimized. A case study of the model was implemented in Fayoum, Egypt. The obtained data was solved numerically by LINGO computer software. The results indicated that the optimal design for management system of solid waste in Fayoum Governorate can yield the optimal solution by installing four centers of collecting waste, one facility for recycling and one landfill strategically located. This showed a linear reduction in the net daily cost as there was increase in the recycling plant.

Govindan, et al. (2021) presented a model of a bi-objective mixed integer linear programming (MILP) for management of medical waste during outbreak of COVID 19 for taking care of both non-infectious waste and infectious waste in unpredictable environment. The objectives of the model simultaneously were minimizing of the overall cost and population exposure risk to pollution. As a result a FGP model was designed to get the solution of the created model. The data used in the study was obtained from Tehran

Municipality Iran. The factors such as separating non-infectious waste from infectious waste were considered during the stages of collection of waste by vehicles, reducing the waiting time for the vehicles entering waste production centers as well as failure for the vehicle to carry infectious waste. The obtained results were useful to managers and decision makers for instance the results indicated that those vehicles whose possibility of failure is low to be allocated to collect hazardous waste and those with higher probability to be assigned to collect non-hazardous wastes.

Mehdi et al.(2021) developed a bi-objective optimization model which aimed at minimizing the cost of the location of facility, transportation organization and the emissions of pollutants of the environment. The uncertainty nature of the problem and the quantity of the generation of waste as a random parameter were considered. A stochastic mathematical programming model with probable constraints was created as a result. The results revealed that by increasing the capacity levels would lead to decrease in the Cost of the location of facility, transportation, organization and the emissions of environmental pollutants.

Onchong'a et al. (2019) conducted a study on the effect of participation of the stakeholders in the management of solid waste's implementation projects in the county of Nyamira, Kenya. A multiple regression analysis was adopted to establish the independent parameters and dependent parameters' relationship. It was discovered that an impressive relationship between participation of stakeholders and management of solid waste projects Implementation in Nyamira County existed. The results showed that involvement of stakeholder was significant. From the basis of the outcome, it was proposed that the county required to continue allowing stakeholders to be involved in the solid waste management implementationcts. The identified stakeholders should be involved from early phases of SWM projects. This is to ensure that their interests and concerns are captured, addressed and incorporated in SWM implementation projects. The involvement of stakeholders was also found to be relevant since it offered assistance in monitoring and evaluating during the implementation of these projects of solid waste management.

Chinchodker, et al. (2017) developed a linear programming model for management of solid waste on ground for waste dumping at Mumbai for reducing the existing cost. The model developed aimed at decreasing the waste transportation cost. The model was presented to satisfy the supply and demand restrictions as it minimizes the total transportation cost using linear programming. The results realized much reduction in the cost of waste transportation than existing one.

Kalu, et al. (2017) worked on a mathematical model for the system of SWM in Nigeria, Aba Metropolis municipality of Abia state. In the study it was observed that the minimum cost of waste management decrease with increase in capacities of the collection centers. The research indicated that by designing the centers of waste collection with maximum capacities minimizes the cost, provided that other factors are held constant.

Soltani et al. (2015) explored multi-criteria decision making for management of solid waste in the municipal involving numerous stakeholders. The study considered MSWM as a process that is complex comprises economic, social and environmental criterion. Besides it provides a process of marking decision in management of solid waste challenges in the municipality such as finding appropriate disposal sites for solid waste and coming up with strategies that will involve as many stakeholders as possible including municipalities, national government, experts, industries and even the public. The research provided the results that showed that analytical hierarchy process is the most effective approach that involves various stakeholders in management of solid waste in municipalities.

Yu et al. (2015) developed a linear and dynamic bi-targeted programming mathematical model for optimizing the long term performance for management of solid waste in the municipal. The model proposed deals simultaneously with productivity of economic calculation and pollution of the environment within various time periods for municipal SWM. The study carried out the optimal changing across the horizon entirely which indicated the accuracy of the model developed. The established mathematical model was solved by LINGO Software and the provided solution was found to be effective in long term planning operation for the system of solid waste in the municipal.

Arena and Di Gregorio (2014) investigated the management planning of municipal solid waste basing on analysis of the movement of the waste. The outcome from the management systems of solid waste in the municipal was described in the study and it was discovered that when combining the materials and substances movement analysis together with environmental methods of assessment is a tool-box that is effective for comparison between the scenario and management of solid waste. In the paper a mathematical model for optimizing the current system of management of solid waste in Tehran and identification of the proper number of facilities for waste transfer and waste processing was developed. The model proposal provides an alternatives way of improving SWM system by reducing the number of the transfer stations and as well as the number of units for waste processing.

Khaiwal et al. (2014) concentrated on analysis of the municipal system of managing solid waste as well as waste minimization methods in India Chandigarh Municipal. This city is located in the Northern parts of India. The relevant information in Chandigarh for the Solid Waste Management methods was scrutinized for conductance of the mentioned research. The important information was received from stakeholders some by use of interviews and also from already stored information relating to transportation of waste and disposal of waste in the registry. It was affirmed that recycling system for management of solid waste in the city was poor. It therefore required environmentally-friendly system for managing solid waste, a good process for decision-making and changing activities of operation. The study proposed that a new framework must be framed on a well-integrated designed system of management of solid waste with cost effectiveness, high rates of recovery and other environmental impacts.

Lohri et al. (2014) analyzed the sustainability of finance in Management of solid waste in the Municipal for system costs and revenues in Ethiopia the city of Bahir Dar. The research examined the data by analyzing the cost-income based from July the year 2009 up to 2011 in June. From analysis it was noted that the system total costs of management of solid waste in Bahir Dar had dramatically increased within the considered period due to increasing costs relating to the transportation of waste such as the cost of collecting waste from the residential, companies, commercial centers and institutions such as stadiums, schools, hospitals, among others. The results obtained from the research indicated that the existence of the correct analysis structure of costs and revenue from system of the Solid Waste Management increased productivity and income in relation to the cost. The results obtained also showed that the cooperation between the private sector and municipality is a sufficient solution for improving the sustainability of the finances of the system of Solid Waste Management.

Sie et al. (2013) conducted a study on the network processing optimality for the managing Solid waste in Malaysia, Iskandar Municipal. A mathematical programming model integrating four cardinal consuming technologies was formulated to enhance the network optimal processing. The developed model provided the combination of the technologies which was the best for disposal and recovery of the solid waste procedures. Furthermore, it was also indicated that the model could be used to project the capacity of the waste facilities and projection of greenhouse gases (GHG) coming from system and provision of solution that is cost effective and as well as optimal for solid waste management in municipality. The research developed a mixed integer linear programming (MILP) mathematical model of profit for municipal SWM system. Basing on the obtained results the best combing technologies for the utilization of solid waste based on allocation of solid waste to value added products was indicated to be recycling (48.44 %), landfill LFG capturing (43.19 %), incinerating (8.34 %) and composting (0.03 %).

Srivastava and Nema (2011) developed a mathematical fuzzy parametric programming mathematical model for identification of treatment facilities and facilities for disposal for management of solid waste capacities, planning and waste flow allocation under uncertainty. Deterministic waste allocation scheme was generated by the formulated model but with no basic provision for support of generating many decision options, desired treatment, facilities for disposal, SWM planning capacities and waste allocation movement in uncertain environment. The model proposed generated waste allocation deterministic schemes without proving bases for the support of generation of many decision options corresponding to the conditions of uncertain system.

Li, et al(2008) explored the uncertainties related to solid waste management operational cost and transportation cost. The study formulated an inexact stochastic quadratic programming model that was expressed in possibility distributions terms and discrete intervals.

Nie et al(2007) did a research on management solid waste planning by considering uncertainty of the problem. As a result, hybrid interval parameter fuzzy robust programming model was developed. The mathematical model was incorporated in the interval parameter optimization framework. Nevertheless, the approach produced interval solutions that did not reflect uncertainty expressed in fuzzy sets.

2.3 Conceptual Framework

Solid waste management system involves the source, sorting, waste recycling, market for waste recycled and disposal. The solid waste producer incurs the cost of transporting waste to the collecting centre. Sorting is done at the sources, after sorting waste to be recycled was taken to recycling facility where transportation and operating costs were incurred and revenue was generated from the sales of recycled waste. The residue at the recycling facility joins the system as part of municipal solid waste therefore the recycling facility does not incur any extra cost of transporting the waste to the landfill. This reduces the nodes considered in the model to three. These are source/ collecting center, landfill and recycling facility/market for recycled waste. The conceptual framework is illustrated in Figure 2.1.

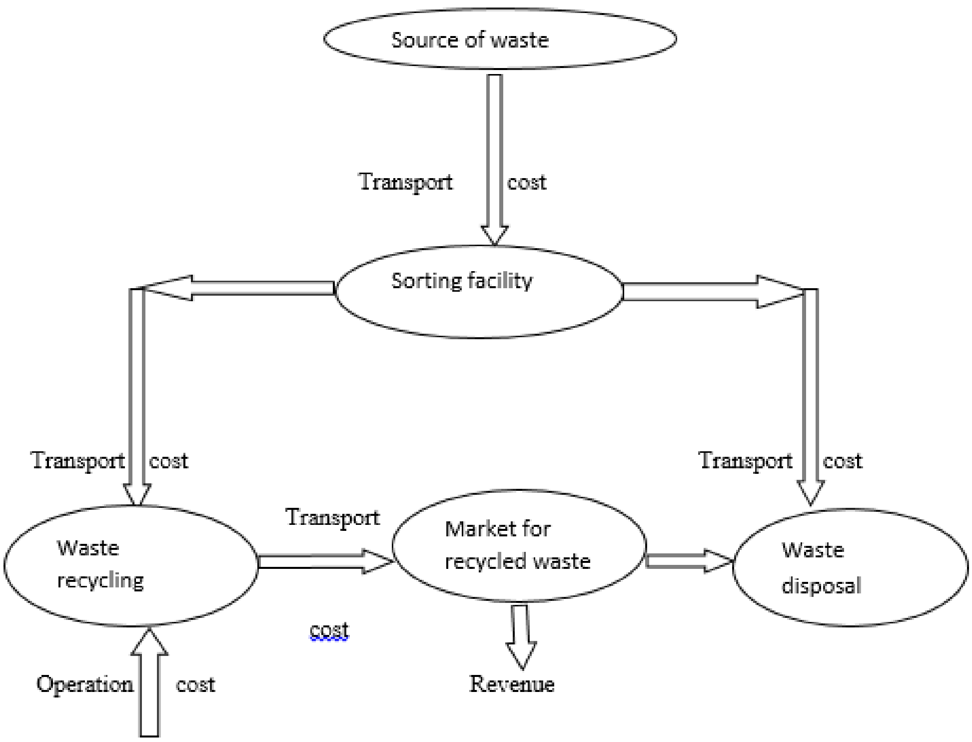

Figure 2.1 Conceptual framework of the model showing components of SWM system

2.4 Identification of Knowledge Gap

The reviewed literature on the solid waste management has not addressed fully the area under the study. For instance Shaban et al. (2022), used mixed integer linear programming model in Fayoum, Egypt. Govindan et al. (2021), presented a bi-objective mixed integer linear programming model of Tehran Municipality, Iran while Mehdi et al. (2021) developed bi-objective stochastic mathematical programming model of solid waste all these among others non is talking about Nyamira Municipality. On the other hand Onchong'a et al. (2019) studied waste management in Nyamira County by adopting multiple regression analysis of stakeholders' involvement. He as well did not capture the aspect of cost and recyclingalso his scope was entire Nyamira county. This study sought to employ scientific approach by developing fuzzy goal programming model focusing on minimizing the cost of waste management and optimizing the profit from recycling.

CHAPTER THREE

METHODOLOGY

3.1 Introduction

In this section a Fuzzy Goal Programming model was formulated. Discussion on how the model was solved analytically using simplex method was demonstrated. The data from Nyamira municipal was used to test the model

3.2 Formulation of the Model

The following variables were used to formulate the fuzzy model.

Indices

The model was formulated by the following indices

i represents source of waste; $i = 1, 2 \ldots n$

j represents disposal and recycling facilities; $j = 1, 2.$

Here $j = 1$ indicates landfill and j=2 represents recycling facilities.

parameters

WTC_{ij} represents the cost of waste transportation from the source i to the facility j.

OC_2 represents the cost of operation of at the recycling facility.

LC represents landfill capacity.

RE_2 represents revenue being generated from recycling facility

RC represents the capacity of recycling facility

W_{ij} represents waste disposal demand

The decision variables

are X_{ij} Quantity of waste from source i to facility j. Using the notations defined above the mathematical model for multi-objective solid waste management (MOSWM) in case of deterministic parameter is formulated as;

$$\text{Minimize } Z_1 = \sum_{i=1}^{n} WTC_{i1}X_{ij} + \sum_{i=1}^{n} WTC_{i2}X_{ij} + \sum_{i=1}^{n} OC_2X_{ij} \tag{3.1}$$

The objective function for minimizing waste management cost. This include transportation cost from collecting center to the landfill and recycling facility and operation cost at the recycling facility.

$$\text{Maximize } Z_2 = \sum_{i=1}^{n} RE_2X_{ij} \tag{3.2}$$

The objective function for the maximizing revenue from recycling facility. Subject to the following constraints

i) Waste Disposal Demand Constraint

The waste disposal demand constraint is

given by

$$\sum_{i=1}^{n}\sum_{j=1}^{2} X_{ij} \geq W_{ij}\text{i} = 1,2\ldots\text{n}, (\text{j} = 1,2) \tag{3.3}$$

This constraint ensures that all the waste generated is removed to ensure clean environment

ii) Landfill Capacity Constraint

The landfill capacity constraint is given by

$$\sum_{i=1}^{n} X_{1i} \leq LC \tag{3.4}$$

The landfill capacity constraints ensures that the waste being taken to the landfill should not exceed the capacity of the landfill.

iii) Recycling Facility Capacity Constraints

The Recycling facility capacity constraint is given;

$$\sum_{i=1}^{n} X_{ij} \leq RC \tag{3.5}$$

The Recycling facility capacity constraints ensures that the waste being taken for recycling should not exceed the capacity of the recycling facility.

In the formulation multi-objective solid waste management (MOSWM) model the parameters are assumed to take deterministic values. But in most of the practical situations these may take imprecise values due to the reasons listed below;

i) The capacity of landfill may vary especially when the model is developed over long period of time.

ii) The generation of waste in Nyamira Municipality is uncertain to the Decision maker.

iii) The price of recycled item may depend on the Decision maker.

Such vagueness in the information is critical which cannot be captured in deterministic problem. Thus the optimal results obtained from deterministic formulations may not sufficiently serve the actual purpose of modeling the problem. Because of this the study considers the model with imprecise information.

In light of the above discussion regarding the possible the MOSWM model, the fuzzy formulation of the problem is done by replacing all deterministic parameters WTC_{ij}, OC_2 and RE_2 with fuzzy parameters $\widetilde{WTC_{\iota}}, \widetilde{OC}_2$ and $\widetilde{RE}_2$ respectively and is therefore expressed as

$$\text{Minimize } \widetilde{Z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{l1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{\iota 2} X_{ij} + \sum_{i=1}^{n} \widetilde{\mathrm{O\,C}}_2 X_{ij} \tag{3.6}$$

$$\text{Maximize } \widetilde{Z}_2 \cong \sum_{i=1}^{n} \widetilde{RE_2} X_{ij} \tag{3.7}$$

Subject to the following constraints

$$\sum_{i=1}^{n}\sum_{j=1}^{2} X_{ij} \gtrsim W_{ij} \tag{3.8}$$

$$\sum_{i=1}^{n} X_{1i} \lesssim \widetilde{LC} \tag{3.9}$$

$$\sum_{i=1}^{n} X_{ij} \lesssim \widetilde{RC} \tag{3.10}$$

and

$$X_{ij} \gtrsim 0 \tag{3.11}$$

3.3 Fuzzy Goal Programming Model for Multi-objective Solid Waste Management

Considering the parameters provided in section (3.1), the decision makers may have fuzzy goals for each of the objectives. To obtain the aspiration level to the fuzzy goals each objective is solved individually for the modified set of system constraints defined in the MOSWM model.

Using Zimmermann's (1978) approach, a fuzzy goal MSWM model is expressed as follows:

$$\text{Minimize } \tilde{Z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2} X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} \precsim g_1 \tag{3.12}$$

$$\text{Maximize } \tilde{Z}_2 \cong \sum_{i=1}^{n} \widetilde{RE}_2 X_{ij} \gtrsim g_2 \tag{3.13}$$

Subject to the following constraints

$$\sum_{i=1}^{n}\sum_{j=1}^{2} X_{ij} \gtrsim W_{ij} \tag{3.14}$$

$$\sum_{i=1}^{n} X_{1i} \lesssim \widetilde{LC} \tag{3.15}$$

$$\sum_{i=1}^{n} X_{ij} \lesssim \widetilde{RC} \tag{3.16}$$

and equation 3.11

where g_1 and g_2 represents aspiration levels for the first goal and second goal respectively. It also means that the Decision Maker (DM) may be satisfied even if it is greater than in case of the first goal and less than for second goal up to a certain tolerance limit.

Considering that the generation of waste can be uncertain, so are landfill and recycling facility capacities as well as budget allocation hence they are said to be fuzzy. The model is modified by substituting W_{ij}, LC and RC_j with d, l and r respectively as follows;

$$\text{Minimize } \tilde{Z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2} X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} \precsim g_1 \tag{3.17}$$

$$\text{Maximize } \widetilde{Z_2} \cong \sum_{i=1}^{n} \widetilde{RE}_2 X_{ij} \gtrsim g_2 \tag{3.18}$$

Subject to following constraints

$$\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} \gtrsim d \tag{3.19}$$

$$\sum_{i=1}^{n} X_{ij} \lesssim \tilde{l} \tag{3.20}$$

$$\sum_{i=1}^{n} X_{ij} \lesssim \tilde{r} \tag{3.21}$$

and equation 3.11

3.4 Defuzzification

There is need to defuzzify the constraints to have corresponding crisp values. The study applies centroid defuzzification method also called centre of area (COA) which is most prevalent defuzzification method (Ross, 2004) whose underlying principle is

$$x^{\times} = \frac{\mu(x)xdx}{\mu(x)dx} \tag{3.22}$$

where $x^{\times}$ is the defuzzified value, $x = (a,b,c)$ indicates the element in $x^{\times}$, and $\mu(x)$ is its associated membership function

Buyukozkan (2012) translated equation (3.22) while defuzzifying a triangular fuzzy number (TFN) by taking the α-cut Set $\widetilde{C_\alpha}$, as follows

$$x^{\times} = \frac{1}{2}\int_0^1 \left(\inf\widetilde{C_\alpha} + \sup\widetilde{C_\alpha}\right) d\alpha \tag{3.23}$$

with α-cut set

$$\widetilde{C_\alpha} = [a + (b-a)\alpha, c - (c-b)\alpha \tag{3.24}$$

Equation (3.24) is further transformed as

$$x^{\times} = \frac{1}{2}\int_0^1 [a + (b-a)\alpha + c - (c-b)\alpha] d\alpha \tag{3.25}$$

$$= \frac{a+c}{2} + \frac{1}{2}\int_0^1 (2b-a-c)\alpha d\alpha \tag{3.26}$$

$$= \frac{a+2b+c}{4} \tag{3.27}$$

Let d, l and r be triangular fuzzy numbers defined by

$$d = \left(d^1, d^2, d^3\right), l = \left(l^1, l^2, l^3\right) \text{ and } r = \left(r^1, r^2, r^3\right) \tag{3.28}$$

with their membership functions as u_d, u_l and u_r respectively.

Using the centroid defuzzification method, we obtain their corresponding defuzified values as

$$d_c = \frac{d^1 + 2d^2 + d^3}{4} \tag{3.29}$$

$$l_c = \frac{l^1 + 2l^2 + l^3}{4} \tag{3.30}$$

$$r_c = \frac{r^1 + 2r^2 + r^3}{4} \tag{3.31}$$

The triangular MF is given by

$$u_d(d_c) = \begin{cases} \frac{d-d^1}{d-d^2} & if \quad d^1 \lesssim d_c \lesssim d^2 \\ \frac{d^3-d}{d^3-d^2} & \text{if} \quad d^3 \lesssim d_c \lesssim d^3 \\ 0 & \text{otherwise} \end{cases} \tag{3.32}$$

$$u_l(l_c) = \begin{cases} \dfrac{l-l^1}{l-t^2} & \text{if} \quad l^1 \lesssim l_c \lesssim l^2 \\ \dfrac{l^3-l}{l^3-l^2} & \text{if} \quad l^3 \lesssim l_c \lesssim l^3 \\ 0 & \text{otherwise} \end{cases} \tag{3.33}$$

$$u_r(r_c) = \begin{cases} \frac{r-r^1}{r-r^2} if r^1 \lesssim r_c \precsim r^2 \\ \frac{r^3-r}{r^3-r^2} if r^3 \lesssim r_c \precsim r^3 \\ 0 \quad otherwise \end{cases} \tag{3.34}$$

After obtaining the difuzzified values d_c, l_c and $r_c \quad \forall \quad i \quad j$ the model (3.17) -(3.18) is subjected to the following constraints

$$\sum_{i=1}^{n} \sum_{j=1}^{2} X_{ij} = d_c, \tag{3.35}$$

$$\sum_{i=1}^{n} X_{ij} \leq l_c, \quad \text{j} = 1 \tag{3.36}$$

$$\sum_{i=1}^{n} X_{ij} \leq r_c, \quad \text{j} = 2 \tag{3.37}$$

and equation (3.11).

In GP, since the goals in most cases are conflicting the decision maker may not be able to achieve their aspirations and therefore deviations may occur such as 'under-achievement' and 'overachievement'. Taking d_i^- to represent under deviation and d_i^+ to represent over deviation, the model (3.17) - (3.18) is expressed as

$$\text{Minimize } \widetilde{Z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} WTC_{i2} X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} + d_1^- - d_1^+ = g_1 \tag{3.38}$$

$$\text{Maximize } \widetilde{Z}_2 \cong \sum_{i=1}^{n} \widetilde{RE}_2 X_{ij} + d_2^- - d_2^+ = g_2 \tag{3.39}$$

Subject the system of constraints from equations (3.35 upto 3.37 and 3.11)

The objective of a GP is to minimize unwanted deviations from objective functions. In this case for the first objective function the decision maker may not wish to overspend and therefore the need to minimize d_1^+ and in the second objective function the revenue should not be below the wish of decision maker therefore the need to minimize d_2^-.

The objective coefficients $\widetilde{WTC}_{i1}, \widetilde{WTC}_{i2}, \widetilde{RE}_2$ and $\widetilde{OC}_2$ are taken as fuzzy numbers implying that the objectives Z_1 and Z_2 must also be fuzzy numbers. Therefore, all fuzzy numbers are triangular fuzzy numbers of the form $a = \left(a^L, a, a^U\right)$, where the superscripts L and U represents lower tolerance and upper tolerance respectively.

Let $Z_1 \cong \left[Z_1^L, Z_1^U\right]$ and $Z_2 \cong \left[Z_2^L, Z_2^U\right]$. To minimize the objective function the lower tolerance corresponds to the aspiration level g_1 while the upper tolerance corresponds the aspiration level g_2 for maximization of the objective function. Therefore, the model is reformulated as

Optimize

$$\mathrm{F} = d_1^+ + d_2^- \tag{3.40}$$

Subject to

$$\sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2} X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} + d_1^- - d_1^+ = g_1 \tag{3.41}$$

$$\sum_{i=1}^{n} \widetilde{RE}_2 X_{i2} + d_2^- - d_2^+ = g_2 \tag{3.42}$$

and other system of constraints from (3.35)to (3.37) and (3.11)

3.5 Construction of Membership Function for FGP

Fuzzy goals are quantified by eliciting the corresponding membership functions on the basis of the achieved values. Thus the linear membership function of each of the objective function is written as

$$\mu Z_1 \cong \begin{cases} 0 & if Z_1 \gtrsim Z_1^U \\ \frac{Z_1^U - Z_1}{Z_1^U - g_1} & if \quad g_1 \lesssim Z_1 \lesssim Z_1^U \\ 1 & if Z_1 \lesssim g_1 \end{cases} \tag{3.43}$$

where Z_1^U is the upper tolerance limit for the first goal and $Z_1^U - g_1$ is the tolerance which is arbitrarily chosen. Also

$$\mu Z_2 \cong \begin{cases} 0 & \text{if } Z_2 \quad Z_2^L \\ \dfrac{Z_2 - Z_2^L}{g_2 - Z_2^L} & \text{if} \quad Z_2^L \lesssim Z_2 \lesssim g_2 \\ 1 & \text{if } Z_2 \gtrsim g_2 \end{cases} \tag{3.44}$$

where Z_2^L is the lower tolerance limit for the second goal and $g_2 - Z_2^L$ is the tolerance which is also arbitrarily chosen.

In fuzzy programming approaches, the highest possible value of membership function is 1 while the lowest is 0

3.6 Solution by Using Simplex Method

The formulated model represented by equations (3.40)-(3.42) and constraints (3.35)-(3.37) and (3.11) was solved by analytical approach using simplex method. The simplex method is an analytical method for finding solution for programming models that are linear by use of pivot variables, tableaus and slack variables as a way of looking for the optimization problem's solution that is desired. The following steps are followed in finding the solution

i) Check that $x_i \geq 0, \forall i = 1, 2 \ldots n$ if not then we replace x_i by $-y_i$ in the given problem so that $y_i \geq 0$

ii) Check if the given problem is maximization. If the problem given is minimization, then we multiply it by -1 in the objective function to convert it to maximization.

iii) We check that $e_i, b_i, l_i, r_j \geq 0$,. If not then we multiply the corresponding constraints by -1 so that $e_i, b_i, l_i, r_j \geq 0$.

iv) Convert all the inequalities of the constraints into equations by introducing fuzzy slack variables.

v) The fuzzy variables $x_1, x_2 \dots x_n$ constituting identity submatrix given the basis $x_i = (x_1, x_2 \dots x_n)$ is the coefficient matrix. The values of $x_1, x_2 \dots x_n$ can be obtained by putting the values of the remaining (N – m) fuzzy non-basic variables equal to zero. Let c_{ij} where $i = 1, 2 \dots n, j = 1, 2 \dots m$ be coefficients of $x_{11}, x_{22} \dots x_{nm}$ respectively in the objective functions.

vi) Construct fuzzy simplex tableau as follows.

Table *3.1*
Simplex algorithm

$\times$	x_1	x_2	$\cdots$	x_n
x_1	x_{11}	x_{12}	$\cdots$	x_{1n}
x_2	x_{21}	x_{22}	$\cdots$	x_{2n}
.				
.				
x_m		x_{m2}		x_{mn}
$D = \sum_{i=1}^{n} c_k x_i$	x_{m1}	$D_2 = C_2$	$\cdots$	$D_N = C_N$

vii) Perform iterations

3.7 Case Study

Nyamira municipality generates about 40 tons of solid waste per day. Out of this figure (40 tons), about only 30 tons per day was being collected (environmental department) leaving about 10 tons of waste uncollected every day. This is why a heaping amount of waste is seen in the streets of the municipality as shown by Figure 6.1

Recovery processes in the municipality mainly include plastic, metallic and rubber waste recycling centers. Further, recyclable waste are mostly locally collected by scavengers *(chokoras)* and then taken to vendors then to recycling/reuse centers.

Currently there is no landfill facility owned by the municipality. The municipality dumps its waste at a privately owned dumpsite for which it pays a fee of ksh. 18,000 per month. The municipality owns a tipper truck and a tractor of capacity 8 tons and 7 tons' respectively. The vehicles collect waste at designated points every weekday. The municipality has contracted casual workers who operates on the vehicles. The tipper truck has 12 casual workers while the tractor has 8 casual workers and 10 casual workers whose role is to manage the landfill. Each casual is paid ksh 800 per day worked for 5 days in a week.

In the last financial year the municipality had the following expenditure as shown in table 3.2

Table *3.2*
Nyamira Municipality Waste Management Expenditure

No.	Item	Cost Breakdown	Total (Ksh)
1	Vehicle Maintenance	i) Tipper truck - @150,000 per month ×12 ii) Tractor - @100,000 per month ×12	3,000,000
2	Salaries	i) Casuals - @800 ×30 casuals ×20 days ×12 months = 5,756,000 ii) Drivers - @25,000 ×4 drivers ×12 months = 1,200,000 iii) Supervisors - @30,000 ×6 supervisors ×12 months = 2,160,000 iv) Other employees in the department - for 450,000 ×12 months = 5,400,000	14,520,000
3	Road Maintenance	@5,000,000 per year	5,000,000
4	Fuel	Tipper and tractor fuel @210 × 33.3 liters per day ×20 days ×12 months = 3,027,310 Supervisors' vehicles	3,027,310 316,000
5	Overall	@3500 ×36	126,000
6	Gumboots	@2000 ×36	72,000
7	Gloves	@1000 ×36	36,000
8	Spades	@1500 ×10	15,000
9	Wheelbarrows	@15,000 ×10	150,000
10	Dustbins	@6000 ×70	420,000
		Total	**26,682,310**

Currently, there is no recycling facility owned by the municipality. Recycling is being done on a small scale in privately owned facilities. These recycling facilities get the waste from collecting centres and others from the dumpsite. Plastic waste was found to be the most recycled solid waste.

3.8 Proposed Fuzzy Goal Programming Mathematical Model

The proposed Fuzzy Goal Programming (FGP) model is a Mathematical Model that optimizes the objectives of minimizing the total cost of SWM, which includes the cost of transporting different types of waste and maximizing revenue collected from recycled waste. The nodes of the transportation network consist of collection, recycling and final disposal node. The proposed (FGP) mathematical model was formulated to determine the establishment of recycling centers at a minimum cost. The study realized that measuring transportation costs per ton is the most preferred in most towns of developing countries. With the current situation in Nyamira municipality where the use of technology to measure waste as it is transported from the waste sources is not available, this study estimated transportation costs in terms of costs per ton of a vehicle from waste collection center i to the facilities.

3.9 Description of the Conceptual Framework of the Proposed Model for the MSWM System

The main focus of the model is to plan the MSW management by defining the refuse flows that have to be sent to recycling centers or to the final disposal sites, from waste sources (residences, markets, schools, restaurants, institutions, hotels etc.). All sorts of wastes produced daily will be moved to collection center i at the expense of generators and some fractions of recyclable/reusable waste are bought/collected and directly taken to vendors/recycling/reuse centers by scavengers. Collection centers are the officially known/adapted points where wastes of a different kind from nearby places (waste sources) are dumped, after which they will be loaded/moved to recycling centers other than to the final disposal site (j=1). Recycling/reusing waste material center is the point where recycling recyclable waste materials such as plastics, rubber and metals are technically feasible. The advantages of recycling waste materials are reducing the amount of waste that reaches the final disposal site. It reduces some of the materials' production costs (e.g., aluminum) and the environmental damage. The final disposal site is the final destination where the waste residue reaches either directly or after passing through different processes. It utilizes a land area to collect the waste with or without separation. Its advantage is that all waste (except hazardous materials) can be dumped without separation.

In this study there are five waste source locations, one landfill location and one recycling facility location with different sections for recycling different wastes that is for papers, plastics and metals. The names for waste sources recycling plants and landfill locations were taken from subdivisions of the municipality. It is important to note that the cost of waste transportation parameter data has been carefully chosen as close to the reality as possible in the municipal. The model assumes that all the waste taken to the recycling facility is recycled and the buyers for the recycled waste come for them at the facility therefore the management does not incur any more cost.

3.10 Data used to test the Mode

Table 3.3 gives locations for sources of waste, recycling plants and landfill. Table 3.4 gives waste sources locations as well as the amount of waste in tones at these sources. Table 3.5 shows distances of waste sources to the landfill. Table 3.6 gives distances of waste sources to the recycling facility. Table 3.7 and table 3.8 gives the capacities of recycling facility and landfill facility respectively. Table 3.9 gives transportation cost to the landfill and recycling facility in triangular fuzzy number (TFN). Table 3.10 gives recycling facility operation cost and the revenue generated in TFN.

***Table** 3.3*
Locations for waste sources, recycling facilities and landfill

	Node type	Locations
1	Waste sources	Township, Miruka, Kebirigo, Nyamaiya, Tinga
2	Recycling plants	Township
3	Landfill	Kemasare

***Table** 3.4*
Distance in kilometers of the landfill from waste sources

	Waste sources locations	Distance (Km)
1	Township	12
2	Miruka	8
3	Kebirigo	22
4	Nyamaiya	5
5	Tinga	20

***Table** 3.5*
Distance in kilometers of the recycling facility from waste sources

	Waste sources locations	Distance (Km)
1	Township	2
2	Miruka	10
3	Kebirigo	10
4	Nyamaiya	17
5	Tinga	8

***Table** 3.6*
The amount of waste at the sources.

	Waste sources locations	Waste amount in tones d:
1	Township	5,475
2	Miruka	2,920
3	Kebirigo	2,555
4	Nyamaiya	1,460
5	Tinga	1,825

***Table** 3.7*
Recycling plants capacity

	Recycling plant	Capacity
1	Township	$(\widehat{2,000}, \widehat{2,500}, \widehat{3,000})$

Table *3.10*

Fuzzy operation cost and revenue at the recycling facility per ton

Operation cost	Revenue
$(\widehat{1000}, \widehat{1200}, \widehat{1400})$	$(\widehat{3600}, \widehat{3800}, \widehat{4000})$

Table *3.8*

Landfill location capacity

	Landfill	$Q_{j=1}$ in tones
1	Kemasare	$(\widehat{13,500}, \widehat{14,000}, \widehat{14,500})$

Table *3.9*

Fuzzy transportation cost from sources to landfill (j=1) and fuzzy +recycling facility (j=2)per ton

i/j	Source/Facility	Kemasare (j=1)	Township (j=2)
1	Township	$(\widehat{1780}, \widehat{1830}, \widehat{1880})$	$(550, \widetilde{6600}, \widehat{650})$
2	Miruka	$\widehat{1720}, \widehat{1770}, \widehat{1820})$	$(700, 750, \widetilde{800})$
3	Kebirigo	$(\widehat{1940}, \widehat{1990}, \widehat{2040})$	$(700, 750, \widehat{800})$
4	Nyamaiya	$(\widehat{1745}, \widehat{1795}, \widehat{1845})$	$(950, 1000, 1050)$
5	Tinga	$(2030, \widehat{2080}, \widehat{2130})$	$(\widehat{600}, 650, 700)$

By using the above information the multi-objective problem of MSWM is formulated by taking X_{ij} and d_c to be deterministic while the rest as TFN. Consequently,

$$
\begin{aligned}
\text{Min}\, Z_1 = {} & (\widetilde{1780}, \widetilde{1830}, \widetilde{1880})x_{1,1} + (\widetilde{1720}, \widetilde{1770}, \widetilde{1820})x_{2,1} + (\widetilde{1940}, \widetilde{1990}, \widetilde{2040})x_{3,1} \\
& + (\widetilde{1745}, \widetilde{1795}, \widetilde{1845})x_{4,1} + (\widetilde{2030}, \widetilde{2080}, \widetilde{2130})x_{5,1} + (\widetilde{550, 600}, \widetilde{650})x_{1,2} \\
& + (\widetilde{700}, \widetilde{750}, \widetilde{800})x_{2,2} + (\overline{700}, \widetilde{750}, \widetilde{800})x_{3,2} + (\widetilde{950},, \widetilde{1000}, \widetilde{1050})x_{4,2} \\
& + (\widetilde{600}, \widetilde{650}, \widetilde{700})x_{5,2} + (\widetilde{1000}, \widetilde{1200}, \widetilde{1400})x_{i,2} \precsim g_1
\end{aligned}
\tag{3.45}
$$

$$
\text{Max}\, Z_2 = \sum_{i=1}^{n} (\widetilde{3600}, \widetilde{3800}, \widetilde{4000})x_{i2} \succsim g_2 \tag{3.46}
$$

Therefore,

$$\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} = 14235, \quad i = 1,2..5, j = 1,2$$
$$\sum_{i=1}^{n} X_{i1} \leq (\widetilde{1,500}, 14, \widetilde{000,14}, 500) \tag{3.47}$$
$$\sum_{i=1}^{n} X_{i2} \leq (\widetilde{2,000}, \widetilde{2,500}, \widetilde{3,000})$$

and equation 3.11

The decision maker assumes that X_{ij} and d_c are deterministic in the model and the right hand side variables are TFNs. Suppose the decision maker desires to spend ksh. 25,977,050 to handle the amount of waste at the sources and he can only tolerate up to ksh. 27,400,550. At the same time the decision maker would like to collect ksh. 10,000,000 from recycling waste but not less than ksh 9,000,000. By applying equation (3.27) for deffuzification, the model is transformed to deterministic form given by

$$\begin{split}\text{Min}\, Z_1 = 1830x_{1,1} + 1770x_{2,1} + 1990x_{3,1} + 1795x_{4,1} + 2080x_{5,1} + 1800x_{1,2} + 1950x_{2,2} \\ +1950x_{3,2} + 2200x_{4,2} + 1850x_{5,2} \precsim 25,977,050\end{split} \tag{3.48}$$

$$\text{Max}\, Z_2 = \sum_{i=1}^{5} 3800x_{i,2} \succsim 10,000,000 \tag{3.49}$$

such that

$$\begin{Bmatrix} x_{1,1} + x_{1,2} = 5,475 \\ x_{2,1} + x_{2,2} = 2,920 \\ x_{3,1} + x_{3,2} = 2,555 \\ x_{4,1} + x_{4,2} = 1,460 \end{Bmatrix} \tag{3.50}$$

$$x_{1,1}+x_{2,1}+x_{3,1}+x_{4,1}+x_{5,1}\leq 14,000 \tag{3.51}$$

$$x_{1,2}+x_{2,2}+x_{3,2}+x_{4,2}+x_{5,2}\leq 2,500 \tag{3.52}$$

By introducing deviations, the model becomes

$$\begin{aligned}\text{Min}\,Z_1 = 1830x_{1,1}+1770x_{2,1}+1990x_{3,1}+1795x_{4,1}+2080x_{5,1}+1800x_{1,2}+1950x_{2,2}\\ +1950x_{3,2}+2200x_{4,2}+1850x_{5,2}+d_1^- - d_1^+ = 25,977,050\end{aligned} \tag{3.53}$$

$$\text{Max}\,Z_2=\sum_{i=1}^{5}3800x_{i,2}+d_2^- - d_2^+ = 10,000,000 \tag{3.54}$$

subject to

constraints (3.50 -3.52) and (3.11)

The model now minimizes the unwanted deviations by optimizing equation (3.40) of deviations

such that

$$\begin{aligned}\text{Min}\,Z_1 = 1830x_{1,1}+1770x_{2,1}+1990x_{3,1}+1795x_{4,1}+2080x_{5,1}+1800x_{1,2}+1950x_{2,2}\\ +1950x_{3,2}+2200x_{4,2}+1850x_{5,2}+d_1^- - d_1^+ = 25,977,050\end{aligned} \tag{3.55}$$

$$\text{Max}\,Z_2=\sum_{i=1}^{5}3800x_{i,2}+d_2^- - d_2^+ = 10,000,000 \tag{3.56}$$

subject to

constraints (3.50 -3..52) and (3.11) The model is then solved by simplex method to obtain the values of deviations and hence obtaining Z_1 and Z_2.

Under these circumstances, the fuzzy- type of the linear membership functions for the objectives functions μZ_1 and μZ_2 is defined for the transportation and operation cost and revenue, respectively, as follows:

$$\mu Z_1 \cong \begin{cases} 0 & \text{if} \quad Z_1 \gtrsim 27,400,550 \\ \frac{27,400,550 - Z_1}{27,400,550 - 25,977,050} & \text{if} \quad 25,977,050 \precsim Z_1 \precsim 27,400,550 \\ 1 & \text{if} \quad Z_1 \precsim 25,977,050 \end{cases} \tag{3.57}$$

$$\mu Z_2 \cong \begin{cases} 0 & \text{if} \quad Z_2 \precsim 9,000,000 \\ \frac{Z_2 - 9,000,000}{10,000,000 - 9,000,000} & \text{if} \quad 9,000,000 \lesssim Z_2 \precsim 10,000,000 \\ 1 & \text{if} \quad Z_2 \succsim 9,000,000 \end{cases} \tag{3.58}$$

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Introduction

This section presented the results from the model formulated in Chapter Three. The developed model for minimizing the net system costs and maximizing the net revenues from recycling was solved using an analytical simplex method in linprog a programme of MATLAB. Sensitivity analysis tests has been conducted on the model.

4.2 Solution of the model

Using the obtained data in the chapter three and the developed model, the parameters were substituted and using the proposed method the results were obtained. To obtain the optimal solution a total of twenty-two iterations were performed.

Table *4.1*
The variables on the model and the translated linprog simplex variables

Model variables	Translated simplex method
$x_{1.1}$	x_1
$x_{2.1}$	x_2
$x_{3.1}$	x_3
$x_{4,1}$	x_4
$x_{5.1}$	x_5
$x_{1.2}$	x_6
x_{22}	x_7
x_{32}	x_{10}
$x_{4.2}$	x_9
$x_{5.2}$	x_{10}
d_1^-	x_{11}
d_1^+	x_{12}
d_2^-	x_{13}
d_2^+	x_{14}

The results of this study involved converting fuzzy constraints into deterministic constraints and solving the formulated FGP model using simplex method.

Table 4.2:

Iterations

Iteration: 1

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9↓	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	25977050	1830	1770	1990	1795	2080	1800	1950	1950	2200	1850	1	-1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	11807.75
x_{22}	M	10000000	0	0	0	0	0	3800	3800	3800	3800	3800	0	0	1	-1	0	0	0	0	0	0	0	1	0	0	0	0	0	2631.58
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	∞
x_{16}	0	2500	0	0	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2500
x_{23}	M	5475	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	∞
x_{24}	M	2920	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	∞
x_{25}	M	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_{26} ←	M	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1460
x_{27}	M	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞

x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_{20}	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		35991285M	1831M	1771M	1991M	1796M	2081M	5601M	5751M	5751M	6001M	5651M	M	-M-1	M-1	-M	0	0	0	0	0	0	0	0	0	0	0	0	0	

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7↓	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	22765050	1830	1770	1990	-405	2080	1800	1950	1950	0	1850	1	-1	0	0	0	0	0	0	0	0	1	0	0	0	0	-2200	0	11674.38
x_{22}	M	4452000	0	0	0	-3800	0	3800	3800	3800	0	3800	0	0	1	-1	0	0	0	0	0	0	0	1	0	0	0	-3800	0	1171.58
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	∞
x_{16}	0	1040	0	0	0	-1	0	1	1	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	0	1040
x_{23}	M	5475	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	∞
x_{24}	M	2920	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2920
x_{25}	M	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_{27}	M	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_{20}←	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	151
min		27229825M	1831M	1771M	1991M	-4205M	2081M	5601M	5751M	5751M	0	5651M	M	-M-1	M-1	-M	0	0	0	0	0	0	0	0	0	0	0	-6001M	0	

Calculation of table elements:

Iteration: 2

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8 ↓	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	22470600	1830	1770	1990	-405	2080	1800	0	1950	0	1850	1	-1	0	0	0	0	0	0	0	-1950	1	0	0	0	0	-2200	0	11523.38
x_{22}	M	3878200	0	0	0	-3800	0	3800	0	3800	0	3800	0	0	1	-1	0	0	0	0	0	-3800	0	1	0	0	0	-3800	0	1020.58

Calculation of table elements:

Iteration: 3

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8 ↓	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	22470600	1830	1770	1990	-405	2080	1800	0	1950	0	1850	1	-1	0	0	0	0	0	0	0	-1950	1	0	0	0	0	-2200	0	11523.38
x_{22}	M	3878200	0	0	0	-3800	0	3800	0	3800	0	3800	0	0	1	-1	0	0	0	0	0	-3800	0	1	0	0	0	-3800	0	1020.58
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	∞
x_{16}	0	889	0	0	0	-1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	-1	0	889
x_{23}	M	5475	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	∞
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25}	M	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	2555
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_{27}	M	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞

x_{17}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19} ←	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	151
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		26361424M	1831M	1771M	1991M	-4205M	2081M	5601M	0	5751M	0	5651M	M	-M-1	M-1	-M	0	0	0	0	0	-5751M	0	0	0	0	0	-6001M	0	

Calculation of table elements:

Iteration: 4

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10} ↓	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	22176150	1830	1770	1990	-405	2080	1800	0	0	0	1850	1	-1	0	0	0	0	0	0	-1950	-1950	1	0	0	0	0	-2200	0	11987.11
x_{22}	M	3304400	0	0	0	-3800	0	3800	0	0	0	3800	0	0	1	-1	0	0	0	0	-3800	-3800	0	1	0	0	0	-3800	0	869.58
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	∞
x_{16}	0	738	0	0	0	-1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	0	-1	0	738
x_{23}	M	5475	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	∞
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25}	M	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_{27}	M	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1825
x_{17}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_{18} ←	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	641
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		25493023M	1831M	1771M	1991M	-4205M	2081M	5601M	0	0	0	5651M	M	-M-1	M-1	-M	0	0	0	0	-5751M	-5751M	0	0	0	0	0	-6001M	0	

Calculation of table elements:

Iteration: 5

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6↓	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{21}	M	20990300	1830	1770	1990	-405	2080	1800	0	0	0	0	1	-1	0	0	0	0	0	-1850	-1950	-1950	1	0	0	0	0	-2200	0	11661.28
x_{22}	M	868600	0	0	0	-3800	0	3800	0	0	0	0	0	0	1	-1	0	0	0	-3800	-3800	-3800	0	1	0	0	0	-3800	0	228.58
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	∞
x_{16} ←	0	97	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	1	0	-1	-1	-1	0	0	0	0	0	-1	0	97
x_{23}	M	5475	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	5475
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25}	M	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_{27}	M	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1462
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		21870732M	1831M	1771M	1991M	-4205M	2081M	5601M	0	0	0	0	M	-M-1	M-1	-M	0	0	0	-5651M	-5751M	-5751M	0	0	0	0	0	-6001M	0	

Calculation of table elements:

Iteration: 6

B	Cb	P	x_1	x_2	x_3	x_4	x_5↓	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21}	M	20815700	1830	1770	1990	1395	2080	0	0	0	0	0	1	-1	0	0	0	-1800	0	-50	-150	-150	1	0	0	0	0	-400	0	10007.5
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	14000	1	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	14000
x_6	0	97	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	1	0	-1	-1	-1	0	0	0	0	0	-1	0	∞
x_{23}	M	5378	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	1	0	0	1	0	0	1	0	∞
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25}	M	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_{27} ←	M	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	1184
x_{17}	0	1365	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	0	0	0	0	0	1	0	∞
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		21327435M	1831M	1771M	1991M	1396M	2081M	0	0	0	0	0	M	-M-1	M-1	-M	0	-5601M	0	-50M	-150M	-150M	0	0	0	0	0	-400M	0	

Calculation of table elements:

Iteration: 7

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18} ↓	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21}	M	18352980	1830	1770	1990	1395	0	0	0	0	0	0	1	-1	0	0	0	-1800	0	2030	-150	-150	1	0	0	0	0	-400	-2080	9040.88
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	12816	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	-1	12816
x_6	0	97	0	0	0	-1	0	1	0	0	0	0	0	0	0	0	0	1	0	-1	-1	-1	0	0	0	0	0	-1	0	-97
x_{23}	M	5378	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	0	1	1	1	0	0	1	0	0	1	0	5378
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25}	M	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	-1184
x_{17}	0	1365	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	0	0	0	0	0	1	0	1365
x_{10} ←	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	641
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		18863531M	1831M	1771M	1991M	1396M	0	0	0	0	0	0	M	-M-1	M-1	-M	0	-5601M	0	2031M	-150M	-150M	0	0	0	0	0	-400M	-2081M	

Calculation of table elements:

Iteration: 8

B	Cb	P	x_1	x_2	x_3↓	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21}	M	17051750	1830	1770	1990	1395	0	0	0	0	0	-2030	1	-1	0	0	0	-1800	0	0	-150	-150	1	0	0	0	0	-400	-2080	8568.72
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	12175	1	1	1	1	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	12175
x_6	0	738	0	0	0	-1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	0	-1	0	∞
x_{23}	M	4737	1	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	1	1	0	0	1	0	0	1	0	∞
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{25} ←	M	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	2404
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	724	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	-1	1	0	1	1	0	0	0	0	0	1	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		17561660M	1831M	1771M	1991M	1396M	0	0	0	0	0	-2031M	M	-M-1	M-1	-M	0	-5601M	0	0	-150M	-150M	0	0	0	0	0	-400M	-2081M	

Calculation of table elements:

Iteration: 9

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19} ↓	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21}	M	12267790	1830	1770	0	1395	0	0	0	0	0	-2030	1	-1	0	0	0	-1800	0	0	1840	-150	1	0	0	0	-1990	-400	-2080	6667.28
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	9771	1	1	0	1	0	0	0	0	0	-1	0	0	0	0	1	0	0	0	1	0	0	0	0	0	-1	0	-1	9771
x_6	0	738	0	0	0	-1	0	1	0	0	0	1	0	0	0	0	0	1	0	0	-1	-1	0	0	0	0	0	-1	0	-738
x_{23}	M	4737	1	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	-1	0	0	1	1	0	0	1	0	0	1	0	4737
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	-2404
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	724	0	0	0	1	0	0	0	0	0	-1	0	0	0	0	0	-1	1	0	1	1	0	0	0	0	0	1	0	724
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_8 ←	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	151
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		12775296M	1831M	1771M	0	1396M	0	0	0	0	0	-2031M	M	-M-1	M-1	-M	0	-5601M	0	0	1841M	-150M	0	0	0	0	-1991M	-400M	-2081M	

Calculation of table elements:

Iteration: 10

B	Cb	P	x_1↓	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21}	M	11989950	1830	1770	0	1395	0	0	0	-1840	0	-2030	1	-1	0	0	0	-1800	0	0	0	-150	1	0	0	0	-1990	-400	-2080	6551.89
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	9620	1	1	0	1	0	0	0	-1	0	-1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	-1	0	-1	9620
x_6	0	889	0	0	0	-1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	-1	0	∞
x_{23} ←	M	4586	1	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	0	0	0	1	0	0	1	0	0	1	0	4586
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	573	0	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	1	0	0	1	0	0	0	0	0	1	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		12497305M	1831M	1771M	0	1396M	0	0	0	-1841M	0	-2031M	M	-M-1	M-1	-M	0	-5601M	0	0	0	-150M	0	0	0	0	-1991M	-400M	-2081M	

Calculation of table elements:

Iteration: 11

B	Cb	P	x_1	x_2↓	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_{21} ←	M	3597570	0	1770	0	-435	0	0	0	-10	0	-200	1	-1	0	0	0	30	0	0	0	-1980	1	0	-1830	0	-1990	-2230	-2080	2032.53
x_{22}	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	5034	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	-1	0	0	-1	0	-1	-1	-1	5034
x_6	0	889	0	0	0	-1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	-1	0	∞
x_1	0	4586	1	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	0	0	0	1	0	0	1	0	0	1	0	∞
x_{24}	M	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	2769
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	573	0	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	1	0	0	1	0	0	0	0	0	1	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		4100339M	0	1771M	0	-435M	0	0	0	-10M	0	-200M	M	-M-1	M-1	-M	0	-3770M	0	0	0	-1981M	0	0	-1831M	0	-1991M	-2231M	-2081M	

Calculation of table elements:

Iteration: 12

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13} ↓	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_2	0	2032.53	0	1	0	-0.25	0	0	0	-0.01	0	-0.11	0	0	0	0	0	0.02	0	0	0	-1.12	0	0	-1.03	0	-1.12	-1.26	-1.18	∞
x_{22} ←	M	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	500000
x_{15}	0	3001.47	0	0	0	0.25	0	0	0	0.01	0	0.11	0	0	0	0	1	0.98	0	0	0	0.12	0	0	0.03	0	0.12	0.26	0.18	∞
x_6	0	889	0	0	0	-1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	-1	0	∞
x_1	0	4586	1	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	0	0	0	1	0	0	1	0	0	1	0	∞
x_{24}	M	736.47	0	0	0	0.25	0	0	0	0.01	0	0.11	0	0	0	0	0	-0.02	0	0	0	0.12	0	0	1.03	1	1.12	1.26	1.18	∞
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17}	0	573	0	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	1	0	0	1	0	0	0	0	0	1	0	∞
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		500736.47M	0	0	0	0.25M	0	0	0	0.01M	0	0.11M	0M	0M-1	M-1	-M	0	-3800.02M	0	0	0	0.12M	-1M	0	0.03M	0	0.12M	0.26M	0.18M	

Calculation of table elements:

Iteration: 13

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26} ↓	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	

x_2	0	2032.53	0	1	0	-0.25	0	0	0	-0.01	0	-0.11	0	0	0	0	0	0.02	0	0	0	-1.12	0	0	-1.03	0	-1.12	-1.26	-1.18	-1613.26
x_{13}	1	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	3001.47	0	0	0	0.25	0	0	0	0.01	0	0.11	0	0	0	0	1	0.98	0	0	0	0.12	0	0	0.03	0	0.12	0.26	0.18	11549.15
x_6	0	889	0	0	0	-1	0	1	0	1	0	1	0	0	0	0	0	1	0	0	0	-1	0	0	0	0	0	-1	0	-889
x_1	0	4586	1	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	0	0	0	1	0	0	1	0	0	1	0	4586
x_{24}	M	736.47	0	0	0	0.25	0	0	0	0.01	0	0.11	0	0	0	0	0	-0.02	0	0	0	0.12	0	0	1.03	1	1.12	1.26	1.18	584.56
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1460
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	∞
x_{17} ←	0	573	0	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	1	0	0	1	0	0	0	0	0	1	0	573
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		736.47M+500000	0	0	0	0.25M	0	0	0	0.01M	0	0.11M	0M	0M-1	0	-1	0	-0.02M-3800	0	0	0	0.12M	-1M	-M+1	0.03M	0	0.12M	0.26M	0.18M	

Calculation of table elements:

Iteration: 14

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10} ↓	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q

			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2754.44	0	1	0	1.01	0	0	0	-1.27	0	-1.37	0	0	0	0	0	-1.24	1.26	0	0	0.14	0	0	-1.03	0	-1.12	0	-1.18	-2006.32
x_{13}	1	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	2852.56	0	0	0	-0.01	0	0	0	0.27	0	0.37	0	0	0	0	1	1.24	-0.26	0	0	-0.14	0	0	0.03	0	0.12	0	0.18	7650.05
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞
x_{24} ←	M	14.56	0	0	0	-1.01	0	0	0	1.27	0	1.37	0	0	0	0	0	1.24	-1.26	0	0	-1.14	0	0	1.03	1	1.12	0	1.18	10.6
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	887	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	1	-1	0	0	-1	0	0	0	0	0	0	0	887
x_5	0	1825	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1825
x_{26}	M	573	0	0	0	1	0	0	0	-1	0	-1	0	0	0	0	0	-1	1	0	0	1	0	0	0	0	0	1	0	-573
x_{18}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	641
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		587.56M+500000	0	0	0	-0.01M	0	0	0	0.27M	0	0.37M	0M	0M-1	0	-1	0	0.24M-3800	-0.26M	0	0	-0.14M	-1M	-M+1	0.03M	0	0.12M	0	0.18M	

Calculation of table elements:

Iteration: 15

B	Cb	P	x_1	x_2	x_3	x_4↓	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{13}	1	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	∞
x_{15}	0	2848.6	0	0	0	0.26	0	0	0	-0.08	0	0	0	0	0	0	1	0.91	0.08	0	0	0.17	0	0	-0.25	-0.27	-0.18	0	-0.14	10900.96
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞
x_{10}	0	10.6	0	0	0	-0.74	0	0	0	0.92	0	1	0	0	0	0	0	0.91	-0.92	0	0	-0.83	0	0	0.75	0.73	0.82	0	0.86	-14.36
x_3	0	2555	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	∞
x_9	0	876.4	0	0	0	0.74	0	0	0	0.08	1	0	0	0	0	0	0	0.09	-0.08	0	0	-0.17	0	0	-0.75	-0.73	-0.82	0	-0.86	1186.43
x_5	0	1814.4	0	0	0	0.74	1	0	0	-0.92	0	0	0	0	0	0	0	-0.91	0.92	0	0	0.83	0	0	-0.75	-0.73	-0.82	0	0.14	2456.26
x_{26}	M	583.6	0	0	0	0.26	0	0	0	-0.08	0	0	0	0	0	0	0	-0.09	0.08	0	0	0.17	0	0	0.75	0.73	0.82	1	0.86	2233.32
x_{18} ←	0	630.4	0	0	0	0.74	0	0	0	-0.92	0	0	0	0	0	0	0	-0.91	0.92	1	0	0.83	0	0	-0.75	-0.73	-0.82	0	-0.86	853.4
x_{19}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		583.6M+500000	0	0	0	0.26M	0	0	0	-0.08M	0	0	0M	0M-1	0	-1	0	-0.09M-3800	0.08M	0	0	0.17M	-1M	-M+1	-0.25M	-0.27M	-0.18M	0	-0.14M	

Calculation of table elements:

Iteration: 16

x_9	0	246	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	-1	-1	0	-1	0	0	0	0	0	0	0	246
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	∞
x_{26}	M	360.6	0	0	0	0	0	0	0	0.25	0	0	0	0	0	0	0	0.23	-0.24	-0.35	0	-0.13	0	0	1.02	0.99	1.11	1	1.16	1454.54
x_4	0	853.4	0	0	0	1	0	0	0	-1.25	0	0	0	0	0	0	0	-1.23	1.24	1.35	0	1.13	0	0	-1.02	-0.99	-1.11	0	-1.16	-683.87
x_{19} ←	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	151
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		360.6M+500000	0	0	0	0	0	0	0	0.25M	0	0	0M	0M-1	0	-1	0	0.23M-3800	-0.24M	-0.35M	0	-0.13M	-1M	-M+1	0.02M	-0.01M	0.11M	0	0.16M	

Calculation of table elements:

Iteration: 17

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16} ↓	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{13}	1	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	-131.58
x_{15}	0	2588.16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1.23	-0.24	-0.35	-0.25	-0.13	0	0	0.02	-0.01	0.11	0	0.16	2111.7
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_9 ←	0	95	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	-1	-1	-1	0	0	0	0	0	0	0	95
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	∞
x_{26}	M	323.16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0.23	-0.24	-0.35	-0.25	-0.13	0	0	1.02	0.99	1.11	1	1.16	1432.28
x_4	0	1041.84	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1.23	1.24	1.35	1.25	1.13	0	0	-1.02	-0.99	-1.11	0	-1.16	-850.05
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		323.16M+500000	0	0	0	0	0	0	0	0	0	0	0M	0M-1	0	-1	0	0.23M-3800	-0.24M	-0.35M	-0.25M	-0.13M	-1M	-M+1	0.02M	-0.01M	0.11M	0	0.16M	

Calculation of table elements:

Iteration: 18

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27} ↓	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	∞
x_{13}	1	861000	0	0	0	0	0	0	0	0	3800	0	0	0	1	-1	0	0	-3800	-3800	-3800	-3800	0	1	0	0	0	0	0	∞
x_{15}	0	2471.73	0	0	0	0	0	0	0	0	-1.23	0	0	0	0	0	1	0	0.98	0.87	0.98	1.1	0	0	0.02	-0.01	0.11	0	0.16	15567.54
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_{16}	0	95	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	-1	-1	-1	0	0	0	0	0	0	0	∞
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	1184
x_{26} ←	M	301.73	0	0	0	0	0	0	0	0	-0.23	0	0	0	0	0	0	0	-0.02	-0.13	-0.02	0.1	0	0	1.02	0.99	1.11	1	1.16	260.38
x_4	0	1158.27	0	0	0	1	0	0	0	0	1.23	0	0	0	0	0	0	0	0.02	0.13	0.02	-0.1	0	0	-1.02	-0.99	-1.11	0	-1.16	-999.57
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		301.73M+861000	0	0	0	0	0	0	0	0	-0.23M+3800	0	0M	0M-1	0	-1	0	0	-0.02M-3800	-0.13M-3800	-0.02M-3800	0.1M-3800	-1M	-M+1	0.02M	-0.01M	0.11M	0	0.16M	

Calculation of table elements:

Iteration: 19

B	Cb	P	x_{1}	x_{2}	x_{3}	x_{4}	x_{5}	x_{6}	x_{7}	x_{8}	x_{9}	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20} ↓	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{2}	0	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	-2769
x_{13}	1	861000	0	0	0	0	0	0	0	0	3800	0	0	0	1	-1	0	0	-3800	-3800	-3800	-3800	0	1	0	0	0	0	0	-226.58
x_{15}	0	2430.38	0	0	0	0	0	0	0	0	-1.19	0	0	0	0	0	1	0	0.99	0.89	0.98	1.09	0	0	-0.12	-0.15	-0.04	-0.14	0	2236.81
x_{6}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_{1}	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_{3}	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_{16}	0	95	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	-1	-1	-1	0	0	0	0	0	0	0	-95
x_{5}	0	923.62	0	0	0	0	1	0	0	0	0.19	0	0	0	0	0	0	0	0.01	-0.89	0.02	-0.09	0	0	-0.88	-0.85	-0.96	-0.86	0	-10672.89
x_{27}	M	260.38	0	0	0	0	0	0	0	0	-0.19	0	0	0	0	0	0	0	-0.01	-0.11	-0.02	0.09	0	0	0.88	0.85	0.96	0.86	1	3008.89
x_{4}	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	-13545311146665402000
x_{8}	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_{7} ←	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	151
min		260.38M+861000	0	0	0	0	0	0	0	0	-0.19M+3800	0	0M	0M-1	0	-1	0	0	-0.01M-3800	-0.11M-3800	-0.02M-3800	0.09M-3800	-1M	-M+1	-0.12M	-0.15M	-0.04M	-0.14M	0	

Calculation of table elements:

Iteration: 20

B	Cb	P	x_{1}	x_{2}	x_{3}	x_{4}	x_{5}	x_{6}	x_{7}	x_{8}	x_{9}	x_{10}	x_{11}	x_{12} ↓	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_{2}	0	2920	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	∞
x_{13}	1	1434800	0	0	0	0	0	0	3800	0	3800	0	0	0	1	-1	0	0	-3800	-3800	-3800	0	0	1	0	0	0	0	0	∞
x_{15}	0	2266.32	0	0	0	0	0	0	-1.09	0	-1.19	0	0	0	0	0	1	0	0.99	0.89	0.98	0	0	0	-0.12	-0.15	-0.04	-0.14	0	4713940
x_{6}	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_{1}	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞

x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_{16}	0	246	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	-1	-1	-1	0	0	0	0	0	0	0	0	∞
x_5	0	936.68	0	0	0	0	1	0	0.09	0	0.19	0	0	0	0	0	0	0	0.01	-0.89	0.02	0	0	0	-0.88	-0.85	-0.96	-0.86	0	-1948300
x_{27} ←	M	247.32	0	0	0	0	0	0	-0.09	0	-0.19	0	0	0	0	0	0	0	-0.01	-0.11	-0.02	0	0	0	0.88	0.85	0.96	0.86	1	514420
x_4	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	∞
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_{20}	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		247.32M+1434800	0	0	0	0	0	0	-0.09M+3800	0	-0.19M+3800	0	0M	0M-1	0	-1	0	0	-0.01M-3800	-0.11M-3800	-0.02M-3800	0	-1M	-M+1	-0.12M	-0.15M	-0.04M	-0.14M	0	

Calculation of table elements:

Iteration: 21

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7↓	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2920	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	2920
x_{13}	1	1434800	0	0	0	0	0	0	3800	0	3800	0	0	0	1	-1	0	0	-3800	-3800	-3800	0	0	1	0	0	0	0	0	377.58
x_{15}	0	2019	0	0	0	0	0	0	-1	0	-1	0	0	0	0	0	1	0	1	1	1	0	0	0	-1	-1	-1	-1	-1	-2019
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	∞
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	∞

x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_{16}	0	246	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0	1	-1	-1	-1	0	0	0	0	0	0	0	0	246
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	∞
x_{12}	1	514420	0	0	0	0	0	0	-180	0	-405	0	-1	1	0	0	0	0	-30	-230	-40	0	-1	0	1830	1770	1990	1795	2080	-2857.89
x_4	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	13545311146665402000
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_{20} ←	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	151
min		1949220	0	0	0	0	0	0	3620	0	3395	0	-1	0	0	-1	0	0	-3830	-4030	-3840	0	-M-1	-M+1	-M+1830	-M+1770	-M+1990	-M+1795	-M+2080	

Calculation of table elements:

Iteration: 22

x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	∞
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	∞
x_{16} ←	0	95	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	-1	-1	-1	0	0	0	0	0	0	0	95
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	∞
x_{12}	1	541600	0	0	0	0	0	0	0	0	-405	0	-1	1	0	0	0	0	-30	-230	-40	180	-1	0	1830	1770	1990	1795	2080	-1337.28
x_4	0	1460	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1460
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	∞
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	∞
min		1402600	0	0	0	0	0	0	0	0	3395	0	-1	0	0	-1	0	0	-3830	-4030	-3840	-3620	-M-1	-M+1	-M+1830	-M+1770	-M+1990	-M+1795	-M+2080	

Calculation of table elements:

Iteration: 23

B	Cb	P	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	x_9	x_{10}	x_{11}	x_{12}	x_{13}	x_{14}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}	x_{20}	x_{21}	x_{22}	x_{23}	x_{24}	x_{25}	x_{26}	x_{27}	Q
			0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	M	M	M	M	M	M	M	
x_2	0	2769	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	1	0	0	0	
x_{13}	1	500000	0	0	0	0	0	0	0	0	0	0	0	0	1	-1	0	-3800	0	0	0	0	0	1	0	0	0	0	0	
x_{15}	0	2265	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	-1	-1	-1	-1	-1	
x_6	0	1462	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	
x_1	0	4013	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	0	0	
x_{10}	0	641	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	
x_3	0	2404	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	1	0	0	
x_9	0	95	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	-1	-1	-1	-1	0	0	0	0	0	0	0	
x_5	0	1184	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	-1	0	0	0	0	0	0	0	0	1	
x_{12}	1	580075	0	0	0	0	0	0	0	0	0	0	-1	1	0	0	0	405	-435	-635	-445	-225	-1	0	1830	1770	1990	1795	2080	
x_4	0	1365	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	-1	1	1	1	1	0	0	0	0	0	1	0	
x_8	0	151	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	
x_7	0	151	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	
min		1080075	0	0	0	0	0	0	0	0	0	0	-1	0	0	-1	0	-3395	-435	-635	-445	-225	-M-1	-M+1	-M+1830	-M+1770	-M+1990	-M+1795	-M+2080	

Calculation of table elements:

Answer:

$F\times = 1080075$

$X\times = (4013; 2769; 2404; 1365; 1184; 1462; 151;$

$151; 95; 641; 0; 580075; 500000; 0; 0; 0; 0; 0)$

Conventions:

From the Simplex Method processes illustrated above the following results shown in the tables 4.3, 4.4 and 4..5 were obtained.

***Table** 4.2*
Model variable solutions

Model variables	Solutions
$x_{1.1}$	4013
$x_{2.1}$	2769
$x_{3,1}$	2404
$x_{4,1}$	1365
$x_{5,1}$	1184
$x_{1.2}$	1462
x_{22}	151
$x_{3,2}$	151
x_{42}	95
$x_{5,2}$	641
d_1^-	0
d_1^+	5810075
d_2^-	500000
d_2^+	0
d_3^+	0
d_3^-	0
d_2^+	0
d_4^+	0
d_4^-	0

***Table** 4.3*
Deviation variable values

Deviations	Values
d_1^-	0
d_1^+	580075
d_2^-	500000
d_2^+	0

***Table** 4.4*
Objective function values

Goal	Objective function	Values
1	Z_1	$26,557,125$
2.	Z_2	$9,500,000$

The obtained results illustrates that if the recycling facility is utilized to full capacity it will provide maximum benefits from generated revenue. The remaining waste is taken to the landfill. The model assumes that any waste generated can be recycled for commercial benefits.

From the results above the cost of managing 14,235 tons of waste is ksh. 26,557,125 which falls below the optimal threshold of ksh 27,400,550 as earlier projected by decision maker in Chapter three (section 3.9). This operational cost consists of the overall costs of waste management from the sources to the landfill, transportation cost of from the source to recycling facility and the cost incurred at recycling facility. This gives a MF of 0.600944068 that indicates a 60% satisfactory level. The triangular membership function for the first objective function is graphically represented by Figure 4.1

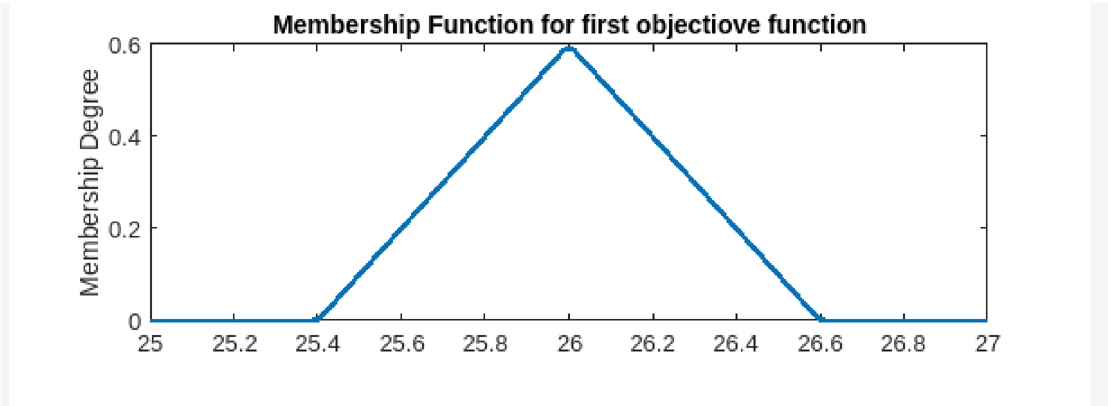

Figure 4.1 First Objective Function

The second objective function intended to maximize the revenue from recycling 2500 tons of waste. The results obtained show that ksh 9,500,000 was generated from the recycling facility which is the maximum revenue that can be generated from recycling indicating MF value of 0.5 giving 50% satisfactory level.

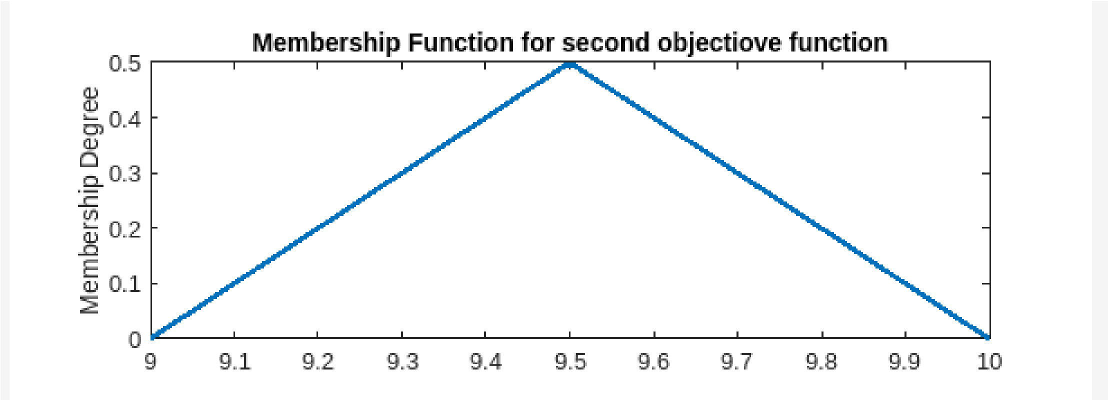

Figure 4.2 Second Objective Function

The results of this study are of immense practical utility since by incorporating the recycling process in waste management, the system will in so many ways reduce the overall cost of waste management from a minimum of ksh 25,977,050 to ksh 16,477,050 which is 36% reduction.

4.3 Sensitivity Analysis

The input data to the FGP model is the key point for the sensitivity analysis of any model. Using the sensitivity analysis based on the optimal solution, the effect of change of parameters can be clearly exemplified. In this section, the sensitivity analysis of operation cost at the recycling facility and recycling limitation will be tested by varying their associated parameters.

4.3.1 Operation Cost Analysis

The sensitivity of the FGP model to the costs of waste management was tested by varying the operational costs of recycling facility as shown in Table 4.6.

Table *4.5*
Different Cases for the varying of the operation Cost

	Case 1	Model cost	Case2
Operational cost	1000	1200	1400
Net Total cost	16,557,125	17,057,125	17,621,225

When the operational cost at the recycling facility was reduced to ksh 1,000 from ksh 1,200 per ton it reduced the net system cost of SWM further to ksh16,557,125 from ksh17,057,125. On the other hand when the operation cost was increased to ksh@,1400 per ton it increased the net system cost to ksh17,621,225. As far as the variable cost for processing facilities is concerned, a change in the variable cost seems to bear the most significant change in the total cost for the SWM system. An increase of variable cost by ksh.200 increased the total cost for the SWM system by 103.30% of the model cost. On the other hand by reducing the variable cost by ksh 200,the FGP model cost will reduce to 97.06% of the model cost. This means that the variable cost parameter is very sensitive therefore care should be taken whenever the decision makers plan for the SWM alternatives. Therefore the operation cost at the recycling facility is directly proportional to the net system cost as represented graphically by Figure 4.3.

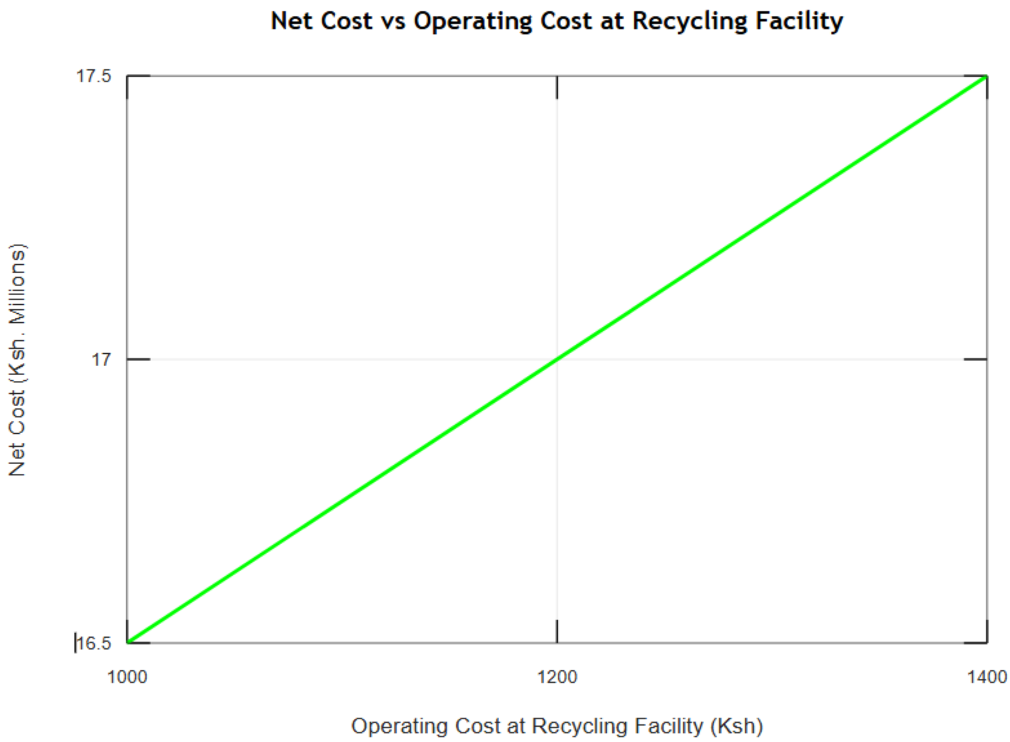

Figure 4.3 Operation Cost vs Total Net Cost

4.3.2 Recycling Limitations Analysis

Table *4.6*
Different cases of the recycling variable capacity

	Case 1	Model capacity	Case 2
Variable capacity	2000	2500	3000
Net system cost	19,144,180	17,057,125	15,131,660

The study considers the sensitivity of changing the capacity of recycling facility to the model formulated as shown in the table 4.7 and illustrated by fig 4.4. Two cases were considered, in the first case the study reduced the recycling capacity by 20% while in the second case the recycling capacity was increased by 20%. The results show the inverse relationship between recycling facility capacity and the net total cost for the SWM system. In the first case, there is an increase of the total cost for the SWM system by 12.23% (Ksh 19,144,180) while on other case there is a decrease of the total cost for the SWM system by 11.28% (Ksh 15,131,660) compared to the base case. Therefore increasing recycling capacity variable reduces the net system cost, that is the recycling facility capacity is inversely proportional to the net system cost.

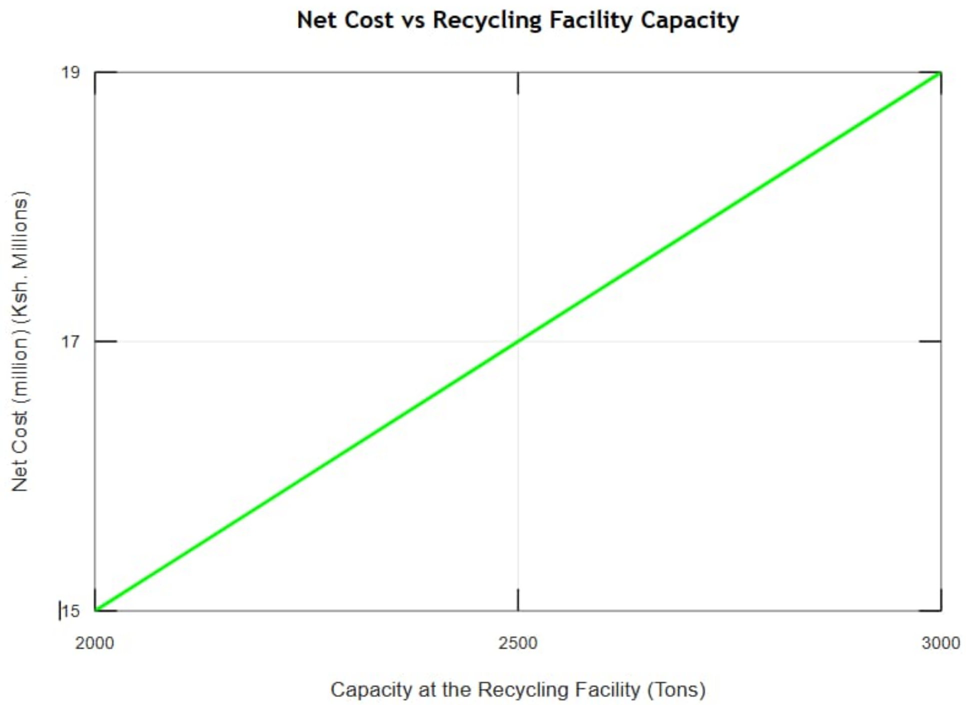

Figure 4.4 Recycling Capacity vs Net Total Cost

CHAPTER FIVE

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

In this chapter a summary of the study is done highlighting important areas covered. Conclusion is made based on the basis of achieved results as per the set objectives. Recommendations are made to policy makers and suggestions for further research is given.

5.2 Summary

In chapter one of this study the paper looked at the introduction where the background of the study was discussed focusing on Nyamira municipality, municipal solid waste, solid waste management and as well as various solid waste management processes. Chapter one goes ahead to talk about mathematical modeling, fuzzy sets and membership function, goal programming as well as fuzzy goal programming. It also contain statement of the problem, objectives of the study, significance, justification, the scope and limitations and the assumptions.

Chapter two took a review of related literature highlighting the contribution of various scholars and positively criticizing the gaps. It has conceptual framework and identifies the gap to be filled.

Chapter Three the study develops the model starting with deterministic parameters then converts it to the fuzzy parameters, it also conducts defuzzification leading to formation of the model represented by equations (3.40-3.42) subject to constraints (3.35-3.37)and (3.11). The study conducts a case study in Nyamira Municipality using the data collected from Environmental services department in the municipal comprising of cost incurred in waste management and the quantity of waste in the municipality.

Chapter four the study applies the data on the model formulated in chapter Three and solves it using simplex method. The obtained results is shown in tables 4.2, 4.3 and 4.4. Sensitivity analysis was done on the recycling facility which suggests that an increase in capacity will decrease the cost even further.

Chapter five looks at the summary, conclusion, recommendations and suggestions for further research. In the appendices the paper has the list of references. It also has figures containing pictures showing the state of waste management in Nyamira Municipality.

5.3 Conclusions

The conclusions of the study

1. A fuzzy goal programming mathematical model was formulated to handle solid waste management in Nyamira Municipality.

2. The model was solved analytically by simplex method. This gave the results for first objective function as ksh. 26,557,125 and second objective function is ksh. 9,500,000. This results yields a membership functions of the first and second objective functions as 0.6 and 0.5 respectively. Incorporating the return from revenue of recycled solid waste reduces the cost of solid waste management from the intial cost of ksh. 26,682,310 to ksh. 17, 057,125 which is 36% reduction.

3. The numerical simulation conducted and as a result graphs were drawn that shown the trend of the outcome of the model.

4. The numerical simulation was done with sensitivity analysis being conducted on the operation cost of the recycling facility and on the capacity of the recycling facility. Varying the operation cost at the recycling facility gives a direct correlation between

net system cost and the cost of operation that is an increase in the cost of operation leads to corresponding increase in the net system cost of solid waste management. The sensitivity analysis was conducted on the capacity of recycling facility showed an indirect relationship between the net system cost and capacity of the recycling facility where an increase in the capacity of recycling facility decreased the net system cost of solid waste management.

Therefore the formulated model has been found to be very effective in the management of solid waste not only in Nyamira Municipality but also other areas.

5.4 Recommendations

The study recommends to the authority in charge of solid waste management in Nyamira Municipality and beyond to consider its findings and implement them in processes of handling of solid waste as follows;

1. Adopt the formulated model in managing solid waste.

2. Incorporating recycling of solid waste as one way of managing solid waste, it does not only generates revenue but also reduces the quantity of was ending at the landfill hence increasing it's lifespan. It will also create job opportunities to the residents both direct and indirect as well as saving on foreign exchange.

3. Collect and store the data on waste management in a retrieval format. this can be retrieved and simulated to monitor the cost of waste management.

4. By increasing the capacity of the recycling facility will go a long way in reducing cost of waste management further. As this is done the authority should ensure that the operation cost should be kept as low as possible.

5.5 Suggestion for Future Studies

The following areas can be considered for future research;

i) The element of time (dynamic element) can be introduced into the model; for instance we can consider activities within time period , where some parameters can change with time t. Planning involves time, and if an application is concerned with a situation that lasts for years, the same types of decisions may have to be made every year. When planning a multi-period horizon (say ,), and there is no change in the data at all from one period to the next, then the optimum solution for the first period found from the static model for that period , will remain optimal for each period in the planning horizon. In most multi-period problems, data changes from period to the next are significant, and the optimum decisions for the various periods may be different, and the sequence of decisions will be interrelated. Designing a dynamic model with the aim of finding a sequence of decisions (one for every period) that is optimal for the planning horizon as a whole, requires reasonably accurate estimates of data for every period of the planning horizon. This is a challenge, but if such data is available, a dynamic model tries to find the entire sequence of interrelated decisions that is optimal for the model over the entire planning horizon.

ii) More waste management alternatives can also be considered in the model such incinerators, compositing and waste to energy recovery.

iii) Conducting sensitivity analysis on transportation cost in the developed FGP model.

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APPENDICES

Appendix I: Photos showing study site

Figure 6.1 a) Pictures showing waste in streets Nyamira municipality.

Figure 6.2 b) Upper Nyamira municipality landfill at Kemasare

Figure 6.3 c) Lower Nyamira municipality landfill at Kemasare

Appendix I1: Questionnaire

QUESTIONNAIRE FOR SOLID WASTE MANAGEMENT SERVICE SURVEY

(WASTE DISPOSAL)

This Questionnaire has been filled by Morris Abere an employee of the department of Environmental social support services as authorized by Morara Mokua (HOD).

1. Introduction

This questionnaire is designed to facilitate the assessment of the current situation of solid waste management service in Nyamira Municipality. The information collected by this questionnaire for all the urban areas in a country, in turn, can be used to evaluate the status of the solid waste management sector in the country.

2. General Information

2.1 Name and address of authority responsible for solid waste management

Nyamira Municipality

2.2 Department responsible for solid waste management

(1) Name, address and telephone of the Department

Environmental Social Support Services

Telephone: 0700122163

(2) Name and telephone of the Head of Department

MR. MORARA MOKUA

Telephone: 0722408029

3. Planning and Development

3.1 Physical characteristics of solid waste

(1) If data on waste characteristics are available, please complete the following table:

Component	% by weight
Paper	15
Plastics	25
Rubber	10
Metal	5
Textile	5
Organic	30
others	10

3.2 Names of the collection centers

i) Township

- Konate
- Sironga

] Township

ii)..Miruka Market..
iii)...Kebirigo Market...
iv)......Nyamaiya..
v)....Tinga Centre.......................................

3.3 Names of the dumpsite/landfill.
i)...Kemasare (dumpsite)........
ii)..

3.5 Names of recycling facilities (if any)
i)...NONE for now but in plan.....
ii)...

4. **Daily estimates of waste at the collection centres**

4.1 from collection centers

Collection center	Amount in tons
Township	15
Miruka	8
kebirigo	~~8~~ 7
Nyamaiya	4
Tinga	5

4.2 waste taken to landfill

Landfill facility	Amount in tons
kemasare	All

4.3 waste taken to Recycling

Recycling center	Amount in tons
NONE	NIL

5 **Finances**

5.1 Distance of landfill from collection centers

Collection center	Distance in km
Township	12
Miruka	8
kebirigo	22
Nyamaiya	5
Tinga	20

5.2 Distance of recycling from collection centers

Collection center	Distance in km
NA	

5.3 capacity of vehicle used in transporting waste

Vehicle type	Capacity in tons
Tipper/Truck)	8
Tractor	7

5.4 Cost of transporting waste from collection centers to landfill

Collection center	Amount in ksh
NOT	AVAILABLE

5.5 Cost of transporting waste from collection centers to recycling

Collection center	Amount in ksh
NA	

5.6 Operation cost at the landfill (includes fixed cost of running facility and Remunerations for workers)

Landfill facility	Amount in ksh
NA.	

5.7Any other expenses towards management of waste.

a) Vehicles maintainance 250,000/= per month (150,000 tipper, 100,000 tract[illegible]
b) Hiring dumpsite 18,000 per month.
c) Road maintainance to dumpsite 5,000,000 per year.
d) Fuel 65,000/= per week
e) Salaries (i) Casuals 30 @ 800/= per day for 20 days per month.
(ii) Drivers 4 @ 25,000 per month.
(iii) Supervisors 6 @ 30,000 per month.
(iv) others in the department cumulatively 450,000/= per month.
f) Other tools.
(i) Overall 36 @ 7500/=
(ii) Gumboots 36 @ 5000/=
(iii) Gloves 36 @ 2500/=
(iv) Spades 10 @ 2500/=
(v) wheel barrows 10 @ 10,000/=
(vi) Dust bins 70 @ 6000/=

These are bought once a year.

N/B These expenses has not been audited.

Appendix III: Research permit

UNIVERSITY OF KABIANGA
ISO 9001:2015 CERTIFIED
OFFICE OF THE DIRECTOR, BOARD OF GRADUATE STUDIES

REF: PGC/AM/003/20

DATE: 15TH JANUARY, 2024

Wilfred Okumu Khafulu,
MAPS Department,
University of Kabianga,
P.O Box 2030- 20200,
KERICHO.

Dear Mr. Khafulu,

RE: CLEARANCE TO COMMENCE FIELD WORK/DATA COLLECTION

I am pleased to inform you that the Board of Graduate Studies has considered and approved your MSc research proposal entitled **"Mathematical Model of Solid Waste Management in Nyamira Municipality using Fuzzy Goal Programming."**

Subsequently the Board has also approved the following supervisors for appointments.

1. Dr. Levi Otanga
2. Dr. Wesley Kirui

You may now proceed to commence field work/data collection on condition that you obtain a research permit from NACOSTI and /or an ethical review permit from a relevant ethics review board.

You are also required to publish one (1) article in a peer reviewed journal, with all your supervisors, before your oral defense of thesis.

You are required to submit through your supervisors, and HoD, progress reports every three months, to the Director, Board of Graduate Studies.

Please note that it is the policy of the University that you complete your studies within three years from the date of registration. Do not hesitate to consult this office in case of any difficulties encountered in the course of your studies.

I wish you all the best in your research and hope that your study will yield original contribution for the betterment of humanity.

Yours Sincerely,

UNIVERSITY OF KABIANGA
15 JAN 2024
BOARD OF GRADUATE STUDIES

Dr. Ronald K. Rop
DIRECTOR, BOARD OF GRADUATE STUDIES.
RKR/lc

cc 1. Dean, SST
2. HOD, MAPS
3. Supervisors

Appendix IV: NACOSTI Research license

REPUBLIC OF KENYA

NATIONAL COMMISSION FOR SCIENCE, TECHNOLOGY & INNOVATION

Ref No: **843583**

Date of Issue: **24/January/2024**

RESEARCH LICENSE

This is to Certify that Mr.. WILFRED OKUMU KHAFULU of University of Kabianga, has been licensed to conduct research as per the provision of the Science, Technology and Innovation Act, 2013 (Rev.2014) in Nyamira on the topic: MATHEMATICAL MODEL OF SOLID WASTE MANAGEMENT IN NYAMIRA MUNICIPALITY USING FUZZY GOAL PROGRAMMING for the period ending : 24/January/2025.

License No: **NACOSTI/P/24/32733**

843583

Applicant Identification Number

Director General
NATIONAL COMMISSION FOR SCIENCE,TECHNOLOGY & INNOVATION

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NOTE: This is a computer generated License. To verify the authenticity of this document, Scan the QR Code using QR scanner application.

See overleaf for conditions

THE SCIENCE, TECHNOLOGY AND INNOVATION ACT, 2013 (Rev. 2014)
Legal Notice No. 108: The Science, Technology and Innovation (Research Licensing) Regulations, 2014

The National Commission for Science, Technology and Innovation, hereafter referred to as the Commission, was the established under the Science, Technology and Innovation Act 2013 (Revised 2014) herein after referred to as the Act. The objective of the Commission shall be to regulate and assure quality in the science, technology and innovation sector and advise the Government in matters related thereto.

CONDITIONS OF THE RESEARCH LICENSE

1. The License is granted subject to provisions of the Constitution of Kenya, the Science, Technology and Innovation Act, and other relevant laws, policies and regulations. Accordingly, the licensee shall adhere to such procedures, standards, code of ethics and guidelines as may be prescribed by regulations made under the Act, or prescribed by provisions of International treaties of which Kenya is a signatory to
2. The research and its related activities as well as outcomes shall be beneficial to the country and shall not in any way;
   i. Endanger national security
   ii. Adversely affect the lives of Kenyans
   iii. Be in contravention of Kenya's international obligations including Biological Weapons Convention (BWC), Comprehensive Nuclear-Test-Ban Treaty Organization (CTBTO), Chemical, Biological, Radiological and Nuclear (CBRN).
   iv. Result in exploitation of intellectual property rights of communities in Kenya
   v. Adversely affect the environment
   vi. Adversely affect the rights of communities
   vii. Endanger public safety and national cohesion
   viii. Plagiarize someone else's work
3. The License is valid for the proposed research, location and specified period.
4. The license any rights thereunder are non-transferable
5. The Commission reserves the right to cancel the research at any time during the research period if in the opinion of the Commission the research is not implemented in conformity with the provisions of the Act or any other written law.
6. The Licensee shall inform the relevant County Director of Education, County Commissioner and County Governor before commencement of the research.
7. Excavation, filming, movement, and collection of specimens are subject to further necessary clearance from relevant Government Agencies.
8. The License does not give authority to transfer research materials.
9. The Commission may monitor and evaluate the licensed research project for the purpose of assessing and evaluating compliance with the conditions of the License.
10. The Licensee shall submit one hard copy, and upload a soft copy of their final report (thesis) onto a platform designated by the Commission within one year of completion of the research.
11. The Commission reserves the right to modify the conditions of the License including cancellation without prior notice.
12. Research, findings and information regarding research systems shall be stored or disseminated, utilized or applied in such a manner as may be prescribed by the Commission from time to time.
13. The Licensee shall disclose to the Commission, the relevant Institutional Scientific and Ethical Review Committee, and the relevant national agencies any inventions and discoveries that are of National strategic importance.
14. The Commission shall have powers to acquire from any person the right in, or to, any scientific innovation, invention or patent of strategic importance to the country.
15. Relevant Institutional Scientific and Ethical Review Committee shall monitor and evaluate the research periodically, and make a report of its findings to the Commission for necessary action.

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